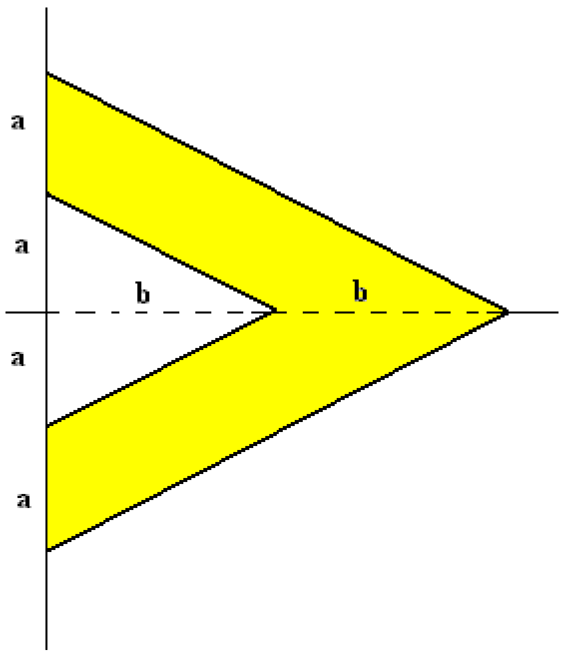
Center of Mass - VI

P-VI. An isosceles-triangular plate is of uniform thickness and constant density. Find the Center of Mass of the plate with the small triangular cutout shown in the figure.



Because it has uniform density and uniform thickness, the mass per unit area is also constant.

We've already solved the triangle problem. This time we have two triangles. Because of the symmetry and the chosen location of the origin, the y-coordinate of the center of mass will be zero. All we have to find is the x-coordinate of the CM.

If the large triangle were a complete triangle, the answer would be obvious from our earlier result. The x-coordinate of the center of mass would be at

$$x_{CM\text{-}original} = \tfrac{1}{3} \cdot 2b = \tfrac{2}{3}b$$

When the small triangle was cut out, the center of mass of the remaining plate moved to the right. The center of mass of the removed plate was at

$$x_{CM\text{-}small} = \tfrac{1}{3}b$$

The center of mass of the original plate can be calculated from the centers of mass of the two component triangles; thus

$$x_{CM\text{-}original} = \tfrac{2}{3}b = \{M_{large} \cdot x_{CM\text{-}large} + M_{small} \cdot x_{CM\text{-}small}\}/\{M_{large} + M_{small}\} \qquad \textbf{Eqn I}$$

As before, we use a constant surface density to substitute (σ • Area) for M. The σ can be factored out of the numerator and denominator and cancelled. The new equation for the x-coordinate of the center of mass, written in terms of the areas, is

$$x_{CM\text{-}original} = \tfrac{2}{3}b = \{A_{large} \cdot x_{CM\text{-}large} + A_{small} \cdot x_{CM\text{-}small}\}/\{A_{large} + A_{small}\}$$

However, we know the areas of the two plates, and we know that $x_{CM\text{-}small} = \tfrac{1}{3}b$.

$\boxed{A_{small} = \tfrac{1}{2}(2a \cdot b) = a \cdot b}$ and $\boxed{A_{large} = \tfrac{1}{2}(4a \cdot 2b) - a \cdot b = 4a \cdot b - a \cdot b = 3a \cdot b}$,

After substituting into Eqn I, factoring a•b from both the numerator and denominator on the right, and then canceling, we find

$$x_{CM\text{-}original} = \tfrac{2}{3}b = \{3x_{CM\text{-}large} + \tfrac{1}{3}b\}/\{3+1\} \text{ or}$$

$\boxed{x_{CM\text{-}large} = (7/9) \cdot b}$; which is farther right than $x_{CM\text{-}original} = \tfrac{2}{3}b = (6/9) \cdot b$.