Finding the Mast Height from the Rising Sun

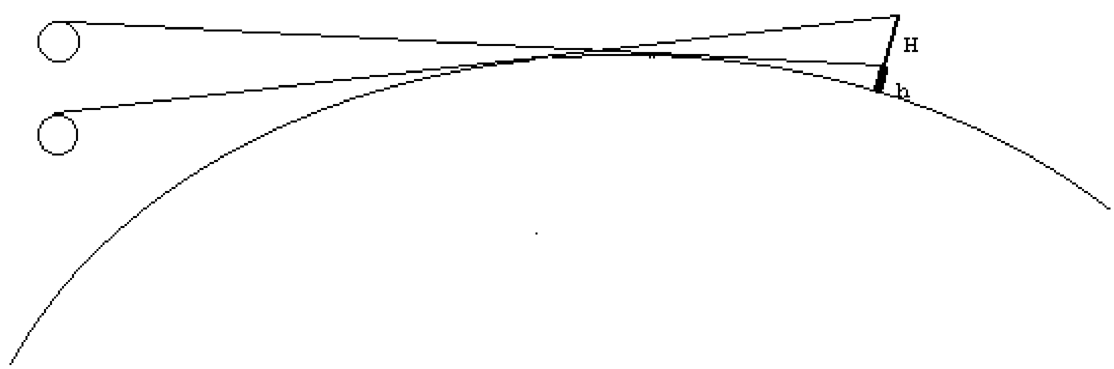
A sailor on the deck of a ship just before sunrise starts a stopwatch the instant sunlight directly illuminates the flag atop the highest mast. He stops the watch the instant he himself sees the top of the sun appear above the horizon. The sailor's eyes are **h** meters above the ocean surface.

Assume a perfectly smooth sea and that the ship is not moving while this measurement is made.

You should use the known relationship between the distance to the horizon, d, and the height of the observer above sea level, h, given by

$$d^2 = 2rh + h^2$$

where r is the radius of the Earth.



What is the vertical height, **H** m, of the top of the mast above sea level if the elapsed time on the stopwatch is **T** seconds?

$d_H - d_h = \sqrt{(2rH - H^2)} - \sqrt{(2rh - h^2)}$: The heights H and h are very, very small compared to Earth's radius, thus

$d_H - d_h = \sqrt{(2rH)} - \sqrt{(2rh)}$

The angle, θ_r in radians, subtended by the two horizon points at the center of the Earth is

$\theta_r = (d_H - d_h) / r$

The ratio of θ_r to f ull circle equals the ratio of T (in seconds) to a full day (in seconds), thus

$\theta_r / 2\pi = T / 86{,}400$ Therefore,

$\theta_r / 2\pi = (d_H - d_h) / 2\pi r = [\sqrt{(2rH)} - \sqrt{(2rh)}] / 2\pi r = \sqrt{(2r)} [\sqrt{H} - \sqrt{h}] / 2\pi r = [\sqrt{H} - \sqrt{h}] / \sqrt{(2r)}\pi = T / 86{,}400$

$H = [\pi T \sqrt{(2r)} / 86{,}400 + \sqrt{h}]^2$

If h = 5 m and T = 30 sec, then H = 37.57 m ≈ 38 m ≈ 40 m ≈ 123 ft ≈ 120 ft