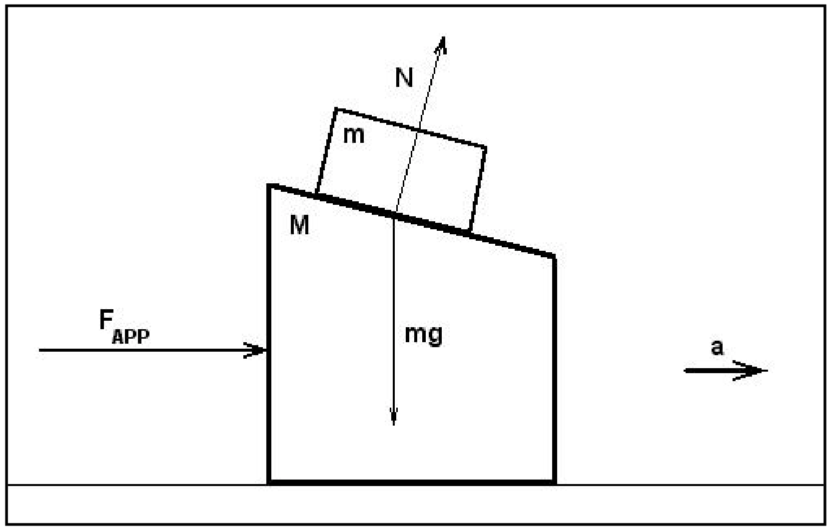
## Two Stacked Blocks Accelerated by a Horizontal Applied Force.

Consider two blocks sitting one on top of the other. Assume all surfaces are frictionless.




**A.** Find the applied force, and resulting acceleration, required to keep the small block stationary atop the large block.

● Some students' first impression is that this is an impossible problem. A more careful analysis shows, however, that it does have a solution and, if looked at from the right point of view, it is easy to solve.

The easiest way to solve this problem is to analyze it from inside the accelerated reference frame.

Note the following facts about the small block:

***i***. It is not moving in the accelerated reference frame. (*Therefore, it can be treated as though it is at equilibrium*.)

***ii***. The normal force is larger than it would be if this small block was sitting on an unaccelerated surface and simply sliding down the slope.

We need to invoke two fictitious forces to satisfy the equilibrium condition in this reference frame.

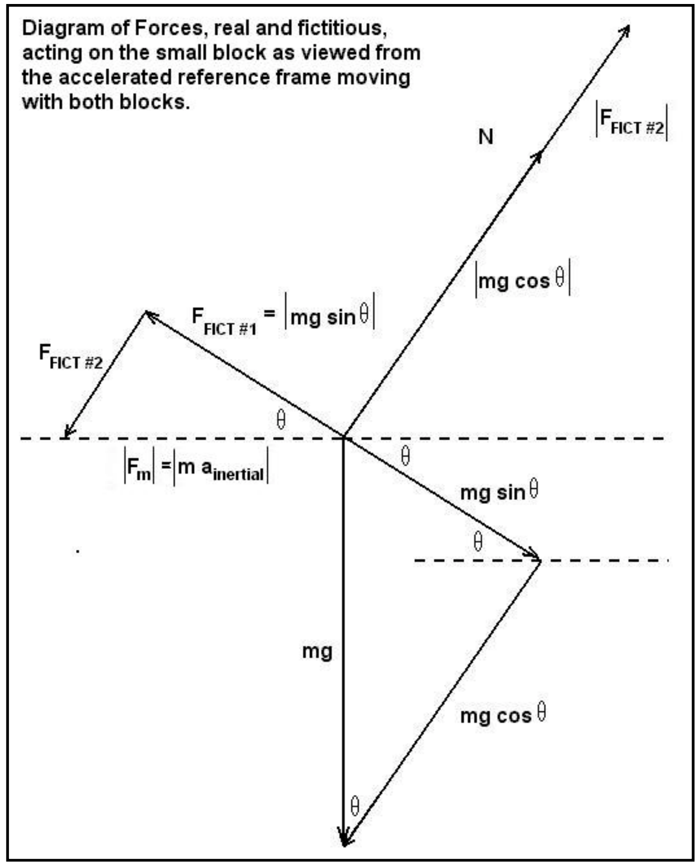
#1) One fictitious force points up the slope and balances the tendency of the block to slide down.

#2) The other fictitious force is perpendicular to the upper surface and balances the increase in the normal force.

Both of these forces are fictitious, of course, and their presence only "seems required" to maintain equilibrium when we are working in an accelerated reference frame. The fact that they are fictitious does not prevent us from using them analytically when working from inside the accelerated reference frame, however.

The vector sum of these two fictitious forces is equal and opposite to that portion of the applied force that is accelerating the small block. Therefore, in principle we know the sum of the two fictitious forces, which will help up find the two components.

● Here is the force diagram for the small block as viewed from the accelerated reference frame.




The vector sum of the two fictitious forces must have the same magnitude as

$$F_m = m\, a_{inertial}$$

Where $a_{inertial}$ is the acceleration of the small block when viewed in the inertial reference frame of the laboratory.

From the small triangle on the left side of the diagram, we can see that

$$m\, a_{inertial} \cos\theta = mg \sin\theta$$

or, that

$$a_{inertial} = g \tan\theta$$

Furthermore, both blocks must have this same acceleration if the small block is not moving with respect to the large block. Since the applied force is accelerating both blocks, we can conclude that

$$F_{app} = F_M + F_m = M\, a_{inertial} + m\, a_{inertial} = (M + m)\, a_{inertial} = (M + m)\, g \tan\theta$$

**B.** Find the normal force and the percentage increase in the normal force compared to the normal force when gravity alone accelerates the small block.

● The normal force from gravity alone equals, as usual and as shown in the diagram, mg cos θ. What we need now is the magnitude of the second fictitious force, $F_{FICT\ \#2}$, so we can add it to mg cos θ to get the new normal force.

Again, we can get this from the small triangle on the left side of the force diagram. Clearly,

$$F_{FICT\ \#2} = m\, a_{inertial} \sin\theta = mg \tan\theta \sin\theta$$

Therefore, the normal force can be calculated as

$$\mathbf{N} = mg\cos\theta + m\, g \tan\theta \sin\theta = mg\,(\cos\theta + \tan\theta \sin\theta) = mg / \cos\theta = mg \sec\theta.$$

● The percentage increase in N over the gravity-only case, is then given by

$$\%Inc = 100\% \bullet [mg \sec\theta - mg\cos\theta] / mg\cos\theta = 100\% \bullet [\sec\theta - \cos\theta] / \cos\theta$$

$$= 100\% \bullet [(1/\cos\theta) - \cos\theta] / \cos\theta = 100\% \bullet [(1 - \cos^2\theta) / \cos\theta] / \cos\theta$$

$$\%Inc = 100\% \bullet [\sin^2\theta / \cos^2\theta] = 100\% \bullet \tan^2\theta$$

## Check of the limiting cases

1. at $\theta = 0^o$,
   $a_{inertial} = g \tan\theta = 0\ m/s^2$, as we would expect.
   $F_{app} = (M + m)\, g \tan\theta = 0$ newtons, as we would expect.
   **N** $= mg / \cos\theta = mg$ newtons, as we would expect.
   $\%Inc = 100\% \bullet \tan^2\theta = 0\%$, as we would expect.

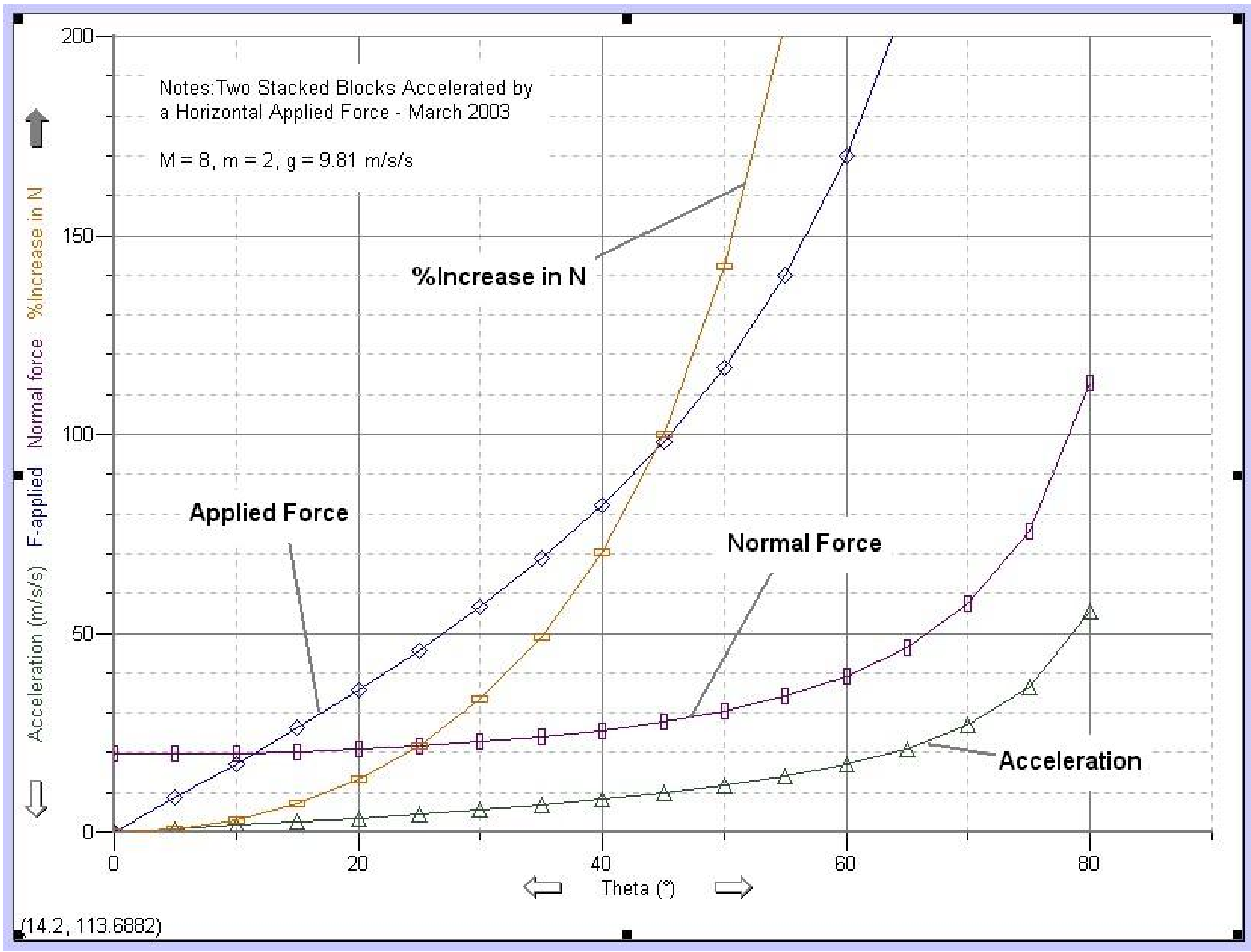
2. at $\theta = 30^o$,
   $a_{inertial} = g \tan\theta = 0.577g\ m/s^2 \approx 5.66\ m/s^2$
   $F_{app} = (M + m)\, g \tan\theta = 0.577\,(M + m)\, g \approx 5.66\,(M + m)$ newtons.
   **N** $= mg / \cos\theta = mg\cos\theta / \cos^2\theta = (4/3)\, mg\cos\theta$; $(4/3)^{rds}$ the gravity-only normal force.
   $\%Inc = 100\% \bullet \tan^2\theta = 33\%$

3. at $\theta = 45^o$,
   $a_{inertial} = g \tan\theta = g\ m/s^2 \approx 9.81\ m/s^2$, as we might have expected.
   $F_{app} = (M + m)\, g \tan\theta = (M + m)\, g$ newtons, as we would expect from the acceleration.
   **N** $= mg / \cos\theta = 2^{½}\, mg$ newtons $= 2\, mg\cos\theta$; twice the gravity-only normal force.
   $\%Inc = 100\% \bullet \tan^2\theta = 100\%$

4. at $\theta = 60^o$,
   $a_{inertial} = g \tan\theta = 3^{½}\, g\ m/s^2 \approx 16.99\ m/s^2$.
   $F_{app} = (M + m)\, g \tan\theta = (M + m)\, 16.99$ newtons.
   **N** $= mg / \cos\theta = mg\cos\theta / \cos^2\theta = 4\, mg\cos\theta$; four times the gravity-only normal force.
   $\%Inc = 100\% \bullet \tan^2\theta = 3^{½} \bullet 100\% = 173\%$

5. at $\theta = 90^o$,
   $a_{inertial} = g \tan\theta =$ undefined, but approaching infinity as $\theta$ approaches $90^o$.
   $F_{app} = (M + m)\, g \tan\theta =$ undefined, but approaching infinity as $\theta$ approaches $90^o$.
   **N** $= mg / \cos\theta =$ undefined, but approaching infinity as $\theta$ approaches $90^o$.
   $\%Inc = 100\% \bullet \tan^2\theta =$ undefined, but approaching infinity as $\theta$ approaches $90^o$.

Everything seems to be in order unless and until someone finds an error in it.




**Note that the percentage increase in the normal force is calculated relative its usual value at the given angle, not to its initial value when the top of the block is horizontal. Thus, at 45-degrees, the percentage increase is 100%. At that angle the normal force is about 28 N. At 45-degrees, the normal force, mg cos θ, would normally be around 14 N.**