Vector Multiplication - Dot Product or Scalar Product

The dot product of two vectors is a scalar quantity. It is defined as follows:

$$\mathbf{a} \bullet \mathbf{b} = |a||b| \cos \theta$$

where θ is the angle between the vectors
when they are placed tail-to-tail

or as

$$\begin{aligned}\mathbf{a} \bullet \mathbf{b} &= [x_a\, \boldsymbol{i} + y_a\, \boldsymbol{j} + z_a\, \boldsymbol{k}] \bullet [x_b\, \boldsymbol{i} + y_b\, \boldsymbol{j} + z_b\, \boldsymbol{k}] \\ &= x_a\, x_b\, \boldsymbol{i} \bullet \boldsymbol{i} + y_a\, y_b\, \boldsymbol{j} \bullet \boldsymbol{j} + z_a\, z_b\, \boldsymbol{k} \bullet \boldsymbol{k} \\ &= x_a\, x_b + y_a\, y_b + z_a\, z_b\end{aligned}$$

The dot product is order independent. The easiest way to remember the multiplication for the dot products of the unit vectors is this

$$\boldsymbol{i} \bullet \boldsymbol{i} = \boldsymbol{j} \bullet \boldsymbol{j} = \boldsymbol{k} \bullet \boldsymbol{k} = 1$$

$$\boldsymbol{i} \bullet \boldsymbol{j} = \boldsymbol{j} \bullet \boldsymbol{i} = \boldsymbol{j} \bullet \boldsymbol{k} = \boldsymbol{k} \bullet \boldsymbol{j} = \boldsymbol{k} \bullet \boldsymbol{i} = \boldsymbol{i} \bullet \boldsymbol{k} = 0$$

The definition in terms of magnitudes and the angle between the vectors works best when working with 2-dimensional vectors. In three dimensions it is often very difficult to find the angle between the two vectors. In three dimensions, or even if two dimensions, it is usually best to work with the component forms of the vectors and use the component definition of the dot product.

Relationships that obey the dot product produce the maximum quantity when the two vectors are parallel and nothing when the vectors are perpendicular to each other. We'll use this form of vector multiplication when we examine the magnetic and electric field vectors passing through surfaces of various shapes and orientations with respect to the field direction.