Vector Multiplication – Sample Problems

You are given the following three vectors:

$\mathbf{A} = 2\,\boldsymbol{i} + 3\,\boldsymbol{j} - 4\,\boldsymbol{k}$
$\mathbf{B} = 3\,\boldsymbol{i} + 4\,\boldsymbol{j} + 2\,\boldsymbol{k}$
$\mathbf{C} = 7\,\boldsymbol{i} + 8\,\boldsymbol{j}$

I. Find **A • B** and **B • A**.

$\mathbf{A} \bullet \mathbf{B} = (2\times3) + (3\times4) + (-4\times2) = 6 + 12 - 8 = 10$

$\mathbf{B} \bullet \mathbf{A} = (3\times2) + (4\times3) + (2\times(-4)) = 6 + 12 - 8 = 10$

Note that **A • B = B • A**

II. Find **A × B** and **B × A**

$\mathbf{A} \times \mathbf{B} = (2\,\boldsymbol{i} + 3\,\boldsymbol{j} - 4\,\boldsymbol{k}) \times (3\,\boldsymbol{i} + 4\,\boldsymbol{j} + 2\,\boldsymbol{k})$
$= 6\,\boldsymbol{i}\times\boldsymbol{i} + 8\,\boldsymbol{i}\times\boldsymbol{j} + 4\,\boldsymbol{i}\times\boldsymbol{k} + 9\,\boldsymbol{j}\times\boldsymbol{i} + 12\,\boldsymbol{j}\times\boldsymbol{j} + 6\,\boldsymbol{j}\times\boldsymbol{k} - 12\,\boldsymbol{k}\times\boldsymbol{i} - 16\,\boldsymbol{k}\times\boldsymbol{j} - 8\,\boldsymbol{k}\times\boldsymbol{k}$
$= 0 + 8\,\boldsymbol{k} - 4\,\boldsymbol{j} - 9\,\boldsymbol{k} + 0 + 6\,\boldsymbol{i} - 12\,\boldsymbol{j} + 16\,\boldsymbol{i} - 0$
$= +16\,\boldsymbol{i} + 6\,\boldsymbol{i} - 12\,\boldsymbol{j} - 4\,\boldsymbol{j} - 9\,\boldsymbol{k} + 8\,\boldsymbol{k}$

$= +22\,\boldsymbol{i} - 16\,\boldsymbol{j} - 1\,\boldsymbol{k}$

$\mathbf{B} \times \mathbf{A} = (3\,\boldsymbol{i} + 4\,\boldsymbol{j} + 2\,\boldsymbol{k}) \times (2\,\boldsymbol{i} + 3\,\boldsymbol{j} - 4\,\boldsymbol{k})$
$= 6\,\boldsymbol{i}\times\boldsymbol{i} + 9\,\boldsymbol{i}\times\boldsymbol{j} - 12\,4\,\boldsymbol{i}\times\boldsymbol{k} + 8\,\boldsymbol{j}\times\boldsymbol{i} + 12\,\boldsymbol{j}\times\boldsymbol{j} - 16\,\boldsymbol{j}\times\boldsymbol{k} + 4\,\boldsymbol{k}\times\boldsymbol{i} + 6\,\boldsymbol{k}\times\boldsymbol{j} - 8\,\boldsymbol{k}\times\boldsymbol{k}$
$= 0 + 9\,\boldsymbol{k} + 12\,\boldsymbol{j} - 8\,\boldsymbol{k} + 0 - 16\,\boldsymbol{i} + 4\,\boldsymbol{j} - 6\,\boldsymbol{i} - 0$
$= -6\,\boldsymbol{i} - 16\,\boldsymbol{i} + 12\,\boldsymbol{j} + 4\,\boldsymbol{j} + 9\,\boldsymbol{k} - 8\,\boldsymbol{k}$

$= -22\,\boldsymbol{i} + 16\,\boldsymbol{j} + 1\,\boldsymbol{k}$

Note that **A × B = – (B × A)**

III. Find **C • (A × B)**

$\mathbf{C} \bullet \mathbf{A} \times \mathbf{B} = (+7\,\boldsymbol{i} + 8\,\boldsymbol{j}) \bullet (+22\,\boldsymbol{i} - 16\,\boldsymbol{j} - 1\,\boldsymbol{k}) = +154 - 128 = +26$

Note that **C • (A × B)** equals the volume of the parallelepiped formed by placing the three vectors at one vertex. Let each vector represent the length and direction of one of the edges that meet at that vertex. **A × B** is a vector whose magnitude equals the area of the base and points perpendicular to the A, B plane. The dot product finds the component of **C** perpendicular to the A, B plane. That component of **C** (*also known as the height*) times the area of the base equals the volume.