Working with Accelerated Reference Frames. I

When working with an accelerated system, we have two options for analyzing the system's behavior. If the system is, or can be viewed as, a single object then it is easiest to view the motion of the system from an external inertial reference frame. We use Newton's Laws of motion to understand the motion and the forces.

However, if we are trying to understand the internal dynamics of a system undergoing acceleration, it is often easier to follow these internal motions from the point of view of the accelerated reference frame. This leads to certain conceptual difficulties.

The biggest difficulty is that we will have to deal with a non-existent force, either as a whole or through its various components. This non-existent force, or the vector sum of all its components, nevertheless has a specific value. It is, in fact, equal in magnitude to the net external force accelerating the system as a whole. Its direction is exactly opposite that of the net external force. Since it has a specific value and direction, we can expect to find its value and use it in Newton's Second Law calculations just as though it were a real force.

Is it fair to use Newton's Second Law, in particular, inside an accelerating reference frame? Strictly speaking, Newton's Laws are not valid inside an accelerating reference frame and Newton never said they were. However, we also know that the error is entirely due to the fact that from inside the accelerating system we cannot formally take account of the accelerating force when trying to understand the internal dynamics. Viewed from the internal reference frame, there is no way for that force to cause the actions we are seeing.

Logic tells us the direction of the external force. Sometimes, we can feel that force. The force we feel seems to act in the wrong direction, however. The appropriate way to handle the non-existent force is to use it as a correction factor for Newton's Second Law.

Simple Problem

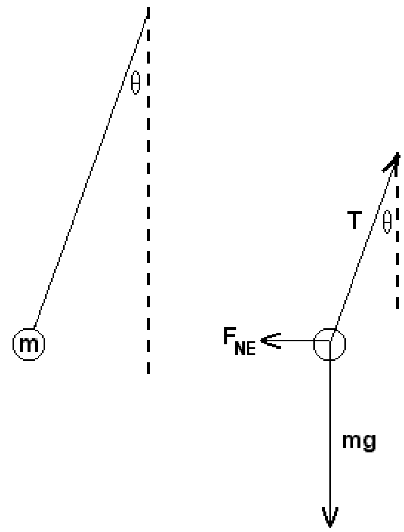
Let's begin with a simple problem that we can all understand.

Imagine you are inside a railroad car. There is a plumb-bob hanging from the ceiling. You can feel that car accelerating and you notice that the plumb bob is not hanging vertically. From the angle of the plumb bob relative to true vertical, calculate the acceleration, $a_{inertial}$, of the railroad car and its contents, in the inertial reference frame of the tracks.

We assume that the track is level and that if the railroad car were not accelerating the plumb bob would hang vertically, i.e. that the track is horizontal. The railroad car is accelerating from left to right in the force diagram that follows.

Let's refer to the non-existent force, in this discussion, as F_{NE}.

Here is the force diagram for the plumb bob.



Inside the accelerated reference frame the system appears to be at equilibrium. The problem is that the only two forces we can identify are the tension, **T**, and the weight, **mg**. We postulate a third force, oriented as shown to establish the apparent equilibrium.

The system is not truly at equilibrium because, as we know, it is accelerating. The choice of the direction of $\mathbf{F}_{NE}$ is also no accident. It points opposite to the direction of the acceleration that we can feel. We are not going to work these problems by pretending that we do not know what is really going on. We are only trying to find insight and perhaps a shortcut to solving this general class of problem.

Note that the tension has a larger magnitude than it would have if the system were not accelerating. If there was no acceleration, then **mg** and **T** would be equal and opposite vectors. In the current situation, however, **T** is larger than **mg** and they are not pointing in opposite directions. (*The changes in magnitude and direction of* **T** *give it a horizontal component; accelerating the bob to the right along with the railroad car.*)

{**Note:** This problem is reminiscent of the circular pendulum problem encountered when investigating centripetal force. In that system, from the point of view of an external inertial reference frame, the centripetal force pointed to the center of the circle. If we had tried analyzing that situation from inside the accelerating reference frame, we would have encountered the exact same force diagram we have here. In that case, we would have called the outward pointing force the centrifugal force. The centrifugal force has the same magnitude but exactly the opposite direction as the centripetal force. We could have used the calculated value of the centrifugal force to determine the correct value of the centripetal force. That is exactly like what we are doing here.}

$$mg = T\cos\theta$$

$$F_{NE} = T\sin\theta$$

$$T = \frac{F_{NE}}{\sin\theta} = \frac{mg}{\cos\theta}$$

$$F_{NE} = mg\tan\theta$$

$$a_{inertial} = g\tan\theta$$

The main advantage of the solution shown left is that we don't need to know the mass of the bob, the mass of the railroad car, or the total applied force on the railroad car. Algebraically, it is similar to the solution for finding the acceleration of the bob from an inertial reference frame. The advantage of working inside the accelerating reference frame is this case is minimal. What this example does show is all the basic steps required to solve a problem from inside an accelerated reference frame.

There are other cases where working inside the accelerated reference frame is by far the simplest and easiest route to the solution.