Name: ______________________________ **Period:** _____ **Due Date:** ___________
Lab Partners: ____

INSTANTANEOUS & AVERAGE VELOCITY - WebAssign

Purpose: To determine an instantaneous velocity as the limit of a series of average velocity measurements over successively shorter time intervals. To observe that the tangent line is the line obtained in the limit as Δt becomes shorter and shorter. To confirm that the derivative equals the slope of the tangent line.

Procedure:

1. Elevate one end of the track about one inch with a small block.

2. Start Logger Pro and open the experiment file Vavg_and_Aavg in the NewLabs folder.

3. Place a post-it ***at the front end*** of the glider. Do not crumple or bend the post-it note. Its width is critical.

4. Place the front of the cart at the 20 cm mark and adjust the first photogate so the flag is about to enter. Then, place the front of the cart at the 160 cm mark and adjust the second photogate so the flag is about to enter. (*You could start with the second photogate at 40 cm, but if you do, fill Table I from the bottom up.*)

5. Start Data Collection mode. Let the cart slide through both photogates from the high end of the track. (D*on't let the cart hit the bottom end of the track*.)

6. Repeat the procedure three times to verify the consistency of your results. Turn off Data Collection and record the data from a representative run in Data Table I. Record **ΔTime 3** only for the trial in which photogate 2 is positioned at 80cm. **ΔTime 1** should be the same for all trials if you are careful.

7. Move the second photogate to the next position on the track and repeat steps #5, #6 and #7.

DATA TABLE I

Gate Positions *(G#1 m, G#2 m)*	**ΔTime 1** *(sec)*	**ΔTime 2** *(sec)*	**ΔTime 3** *(sec)*
0.200, 1.600	_______	_______	
0.200, 1.400	_______	_______	
0.200, 1.200	_______	_______	
0.200, 1.000	_______	_______	
0.200, 0.800	_______	_______	_______
0.200, 0.600	_______	_______	
0.200, 0.400	_______	_______	

Elapsed Time: The elapsed time is the time interval from the moment the flag enters the first photogate until the moment it enters the second photogate. The time interval is the sum, ΔTime 1 + ΔTime 2.

Table II

	Gate Positions *(G#1 m, G#2 m)*	t = Elapsed Time = $t_{Gate\ 2} - t_{Gate\ 1} = t_{Gate\ 2} - 0$ (ΔTime 1 + ΔTime 2) *(sec)*	
#1	*0.200, 1.600*	______________	
#2	*0.200, 1.400*	______________	
#3	*0.200, 1.200*	______________	
#4	*0.200, 1.000*	______________	
#5	*0.200, 0.800*	______________	$= t_r$
#6	*0.200, 0.600*	______________	
#7	*0.200, 0.400*	______________	
#8	*0.200*	*0.000*	

Analysis:

1. Use ***Graphical Analysis*** to make a graph of 2nd gate position vs elapsed time. Call it **Graph #1**.

2. Click at the top of the first data column. Enter ***time*** for the name (**t** for short) and ***sec*** for the units. In the second column enter ***position*** for the name (**x** for short) and ***meters*** for the unit. Enter the elapsed time in the time column (ΔTime 1 + ΔTime 2), and the second gate position in the position column. **Graph #1** is a **Position vs Time** graph. Add an eighth point at time **t** = 0.000 sec; the first gate position is x = 0.200 m.

3. Click on the graph window. Then click on the ***Options-GraphOptions*** menu. Click on ***Connect Points*** to turn ***OFF*** connecting lines.

4. Select ***Analyze-CurveFit*** from the menu. The points in **Graph #1** should fall on the line described by the position equation; $x = x_0 + v_0{*}t + ½\ a{*}t^2$. Click on the ***Define Function*** button and edit the box to read **0.200 + B*t + 0.5*A*t^2.** Click on ***Try Fit***. You should get a parabolic curve that fits the points. Click ***OK***. Drag the box containing the function away from the curve. (*Note: 0.200 m is the starting point for the measurements, i.e.,* x_0*. The coefficient of* **t**, **B,** *is the instantaneous initial velocity,* v_0*, and the coefficient of* t^2 *is one-half of the acceleration; thus,* ***A*** *= acceleration =* a*.*) Move the square brackets to the edges.

5. Include your name, your class period, the lab number, and the graph name in the text box. Change the coordinates of the lower left-hand corner of **Graph #1** to (–1.000 s , 0.00 m). Set t_{MAX} = 3.0 sec. Set x_{MAX} to 2.0 m. On the ***Options-GraphOptions*** menu set Minor Tick Style to "Dashed". It is also a good idea to deselect (*remove the check mark*) the "Show Uncertainty" option in the curve fit box's pop-up menu.

6. Print the entire screen, Data, Text and Graph. Then print the Graph by itself. Make sure the printouts are all in landscape mode to make them as large as possible. You might print a couple of copies of the graph alone just in case you mess up in section 7. Write your name on the graph alone, as there is no text box.

7. You will now attempt to draw the secant lines and the tangent line to a **reference point** (t_r, x_r) on the parabola. The value of t_r varies from group to group, but the value of x_r, 0.800 meters, is the same for all groups. Drawing these lines needs to be done carefully, so wait for specific instructions. The graph will be too messy to read if you make more lines than are needed or if you draw them over too wide a range.

Each secant line starts at a data point and then passes through the reference point, whose position is 0.800 m, and then continues for about two inches farther, but NOT to the edge of the graph. Make sure the secant lines only extend across the parabolic curve at x_r = 0.800 m. If you cross the curve at the other point your drawing will be hard to read and it will be difficult to draw the correct tangent line and measure its slope.

With a pencil and a ruler, draw seven straight secant lines each passing between the pairs of position points defined below. The Line connecting each pair of points is called a secant line. If you like color-coding, then draw the secant lines from the left in one color, and from the right in another. Each secant line should be extended about two inches past the x_r = 0.800 m point. Note, all secant lines pass through the point where x_r = 0.800 m. This is the position, for which we hope to find v_r, the true instantaneous velocity, at coordinates:

(t_r sec, x_r = 0.800 m) = (__________s, 0.800 m) (*Times vary by group, but should be close to* $t_r \approx$ *1.7 sec*).

Make sure your drawing has all the lines passing through this point. *(Time is the horizontal axis in this graph. Position is the vertical axis. Think about what you are doing before you start doing it.)*

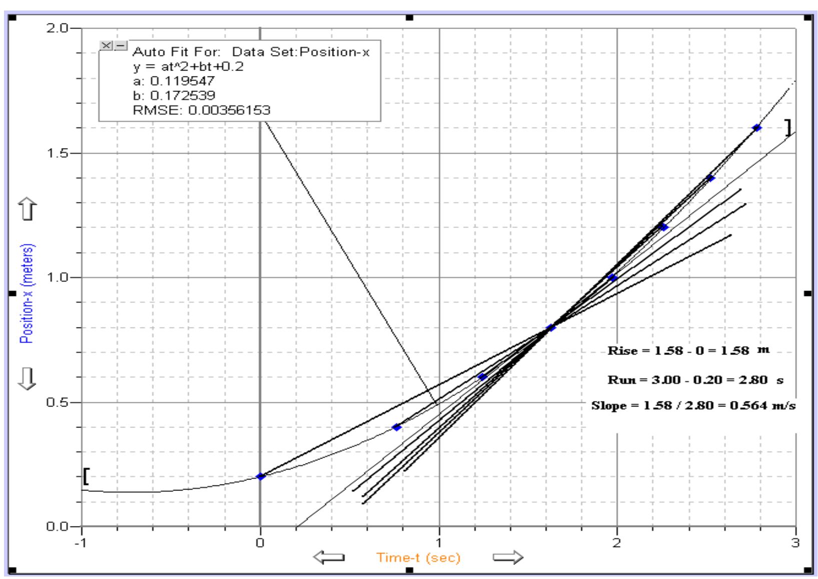
The four lines on the left side of the list below come from the **upper right** section of the graph. These drawn lines come down through the reference point, and are extended down to the left. The three lines on the right side of the list come from the **lower left** section of the graph. These drawn lines come up through the reference point, and are extended up to the right. The tangent line also goes through the reference point, but it should be below the parabola and above all the secant extensions, as shown on the sample graph below. *(If you mess up the graph, reprint it and try again. This needs to be done carefully and correctly.)* Make sure the tangent line extends to the edges of the graph. The points in the list are defined only by their positions. Look to the left side of the graph, at the vertical axis, when you need to locate one of these points.

1.600→0.800   *1.400→0.800*   *1.200→0.800*   *1.000→0.800*   *0.200→0.800*   *0.400→0.800*   *0.600→0.800*

The tangent line should be the only drawn-line that extends to the edges of the graph. This sets it apart from the extended secant lines, and makes it easier both to find it and to calculate its slope. Or, use a different color.

Notice that the tangent line is constrained to fall between the secant line extensions and the parabolic curve. This narrow window makes it easier to draw an accurate tangent line. The tangent line must go through the reference point and both gaps in the secant line extensions.

Below, you will compare the slope of the tangent line to the measured velocity at x_r = 0.800 m, and to the velocity predicted by the velocity equation at time t_r and position x_r = 0.800 m.



8. Calculate the slope of each of the seven, secant lines drawn on **Graph #1 (***Section* **9)** plus the slope of the tangent line drawn through the point at ($\mathbf{t}_r$, $\boldsymbol{x}_r$ = 0.800) (*Section* **10**). The information you need to calculate both $\Delta \boldsymbol{x}$ and $\Delta \mathbf{t}$ for the secant lines is contained in Table II. Extensive instructions and sample calculations for completing Table III are given below in sections **9** – **12**.

Table III

$x_r \rightarrow x$	$\Delta x = x - x_r$	$\Delta t = t - t_r$	*Slope* = $\Delta x/\Delta t$ = average velocity
0.800→1.600	+________	+________	+________ *m/s*
0.800→1.400	+________	+________	+________ *m/s*
0.800→1.200	+________	+________	+________ *m/s*
0.800→1.000	+________	+________	+________ *m/s*

Manual estimate of the Slope of the tangent line
at $\boldsymbol{x}$ = 0.800 *meters* (From **Graph #1**) = v_{manual} = +________ *m/s*

Calculus based estimate of the Slope of the tangent line *at* $\boldsymbol{x}$ = 0.800 *meters*
(From the derivative of the position equation of Graph #1) = $v_{Calculus}$ = +________ *m/s*

Direct measurement of the instantaneous velocity at $\boldsymbol{x}$ = 0.800 *meters*
(From the post-it note width and the measured Δ**Time 3**) = $v_{measured}$ = +________ *m/s*

0.800→0.600	–________	–________	+________ *m/s*
0.800→0.400	–________	–________	+________ *m/s*
0.800→0.200	–________	–________	+________ *m/s*

9. The slopes of the secant lines drawn on **Graph #1** equal the displacement $\Delta \mathbf{x} = (x - x_r)$ divided by the time interval $\Delta \mathbf{t} = (\mathbf{t} - \mathbf{t}_r)$. $\Delta \mathbf{t}$ is most simply calculated as the difference in elapsed times at the two positions. Adjust the signs accordingly. In the sample graph on the previous page, the elapsed time at 0.800 m was 1.628 sec. The elapsed time at 0.200 m was 0.000 sec, therefore, the $\Delta \mathbf{t}$ between 0.200 m and 0.800 m is 0.000 – 1.628 = –1.628 sec. $\Delta \mathbf{x}$ is (0.200 – 0.800). Thus, the **slope of the secant line** between 0.200 m and 0.800 m = (0.200 – 0.800) / (0 – 1.628) = +0.369 m/s. The **average velocity** is therefore +0.369 m/s.

Here is another example. The elapsed times at 1.600 m and 0.800 m are 2.780 sec and 1.628 sec, respectively. Thus, $\Delta \mathbf{t}$ = (2.780 – 1.628) = +1.152 sec. $\Delta \mathbf{x}$ is (1.600 – 0.800). Thus, the **slope of the secant line** between 1.600 m and 0.800 m = (1.600 – 0.800) / 1.152 = +0.694 m/s = **average velocity**.

Complete section 9 and include the slope of the tangent line in Table III. It should lie between the average velocities calculated from the slopes of the secant lines. See section 9 for instructions

10. On the graph, use a ruler to draw the tangent to the curve at the point ($\mathbf{t}_r$, $\boldsymbol{x}_r$ = 0.800 m). Extend the tangent line to the bottom and right edges of the graph. Measure the rise and the run and calculate the slope of the tangent line. This is your manual estimate of the instantaneous velocity, v_r, of the cart at ($\mathbf{t}_r$, $\boldsymbol{x}_r$ = 0.800).

Instantaneous Velocity at point ($\mathbf{t}_r$, $\boldsymbol{x}_r$ = 0.800 m) = Slope of tangent = v_{manual} = ____________ m/s

Enter this value in Table III above.

11. The slope of the tangent line (the instantaneous velocity) can also be found by taking the derivative of the **position vs time equation**. ***Graphical Analysis*** gave you this equation when you found the curve fit for **Graph #1**. The derivative is a mathematical operation that converts any equation into a new equation that gives the slope of any line tangent to the original equation. The derivative of the position equation turns out to be the equation we have come to know as the velocity equation.

Instantaneous Velocity (at any time t) = $\mathbf{{}^{d}/_{dt}\,(\boldsymbol{x}) = \boldsymbol{v} = {}^{d}/_{dt}\,(0.200 + B\,t + \frac{1}{2}\,A\,t^2) = \underline{B + A\,t}}$

Consult the equation coefficients on your printed graph: $\mathbf{B} = \boldsymbol{v_0} =$ __________ and $\mathbf{A} = \boldsymbol{a} =$ __________

As the cart enters the 2nd photogate at 0.800 m, find the elapsed time $\mathbf{t}_r$ from your data table. Then find the instantaneous velocity at that time using the velocity equation derived from the position equation.

Instantaneous Velocity at point ($\mathbf{t}_r$, $\boldsymbol{x}_r = 0.800$ m) = $\boldsymbol{v}_{Calculus}$ = __________________

Enter this calculus-based estimate of the instantaneous velocity, $\boldsymbol{v_r}$, at ($\mathbf{t}_r$, $\boldsymbol{x}_r = 0.800$ m) into Table III.

The slope of the hand-drawn tangent line on your graph and the calculated derivative give you two independent estimates for the instantaneous velocity at ($\mathbf{t}_r$, $\boldsymbol{x}_r = 0.800$ m).

These two estimates should agree. Do they? ____________ (*If not, find your error or see the instructor.*)

12. You measured Δ**Time 3** when the 2nd photogate was located at $\boldsymbol{x}_r = 0.800$ m, so you also have an opportunity to directly measure the instantaneous velocity at that position. We'll call this the measured **Instantaneous Velocity** at ($\mathbf{t}_r$, $\boldsymbol{x}_r = 0.800$ m). It is simply the width of the post-it note divided by Δ**Time 3**.

Width of post-it note = d = ____________ mm = ____________ m

Instantaneous Velocity at point ($\mathbf{t}_r$, $\boldsymbol{x}_r = 0.800$ m) = $\boldsymbol{v}_{measured}$ = d / Δ**Time 3** = ___________ m/s

Enter this measured estimate of the instantaneous velocity, $\boldsymbol{v_r}$, at ($\mathbf{t}_r$, $\boldsymbol{x}_r = 0.800$ m) into Table III.

All three estimates of the instantaneous velocity at point ($\mathbf{t}_r$, $\boldsymbol{x}_r = 0.800$ m) should agree. Any differences should be very small. Do they agree? ___________ (*If not, find your error or see the instructor*.)

(Note: *this measured estimate of $\boldsymbol{v}_r$ will probably be a little high, because the cart does travel is little bit further than 0.800 m while we make the measurement. That is a bit of error we may just have to live with*.)

13. To make a quantitative comparison among the three estimates of $\boldsymbol{v_r}$ at point ($\mathbf{t}_r$, $\boldsymbol{x}_r = 0.800$ m), you will compute the percent difference between each of them and the mean value of the three estimates. Use the mean value as the reference value in the percent difference calculation.

First find the mean value = $\boldsymbol{v}_{mean} = (\boldsymbol{v}_{manual} + \boldsymbol{v}_{Calculus} + \boldsymbol{v}_{measured}) / 3$ = ________________ m/s

%Difference (in $\boldsymbol{v}_{manual}$) = $100\%\,|\,\boldsymbol{v}_{manual} - \boldsymbol{v}_{mean}\,|\,/\,\boldsymbol{v}_{mean}$ = _____________________ %

%Difference (in $\boldsymbol{v}_{Calculus}$) = $100\%\,|\,\boldsymbol{v}_{Calculus} - \boldsymbol{v}_{mean}\,|\,/\,\boldsymbol{v}_{mean}$ = _____________________ %

%Difference (in $\boldsymbol{v}_{measured}$) = $100\%\,|\,\boldsymbol{v}_{measured} - \boldsymbol{v}_{mean}\,|\,/\,\boldsymbol{v}_{mean}$ = ____________________ %

14. Can you conclude anything meaningful about these three methods of finding the instantaneous velocity?

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15. Here is the rule describing the relationship between the average velocity and the instantaneous velocity:

"The instantaneous velocity is the limit of the average velocity as the time interval approaches zero."

Look at the trend in Table III. As the magnitude of Δx decreases toward zero, what happens to the average velocity (*the slope of the secant line*)? The trend in the slopes, as Δx approaches zero, indicates that the slopes are approaching closer to the true value of the instantaneous velocity, v_r, (*as indicated by our three estimates for* v_r). To examine the approach of the average velocity to the instantaneous velocity a little more closely, construct a second graph using the data in Table III. This will be a graph of Average Velocity ($\Delta x/\Delta t$) vs Time Interval (Δt). For comparison purposes, the three estimates of v_r should be graphed at time $\Delta t = 0$, since the elapsed time from the reference point to itself must necessarily be zero seconds. (Graph $-2 \le \Delta t \le +2$)

Call this **Graph #2**. Fit the data to the linear function, y = m**t** + b. This line should NOT go through the origin. The y-intercept should equal the instantaneous velocity, v_r, of the cart as it reaches $x_r = 0.800$ m.

m = ______________ m/s^2, **b** = _______________ m/s

The first theoretical equation you might think of using is the velocity equation. That is $v = v_0 + a\,t$. Unfortunately, the velocity equation only applies to instantaneous velocities. Also, note that **m** is not equal to the acceleration you found in section **10**. So how are we to interpret this graph?

What we actually have are three independent estimates of the instantaneous velocity, v_r, at Δt = zero; v_{manual}, $v_{Calculus}$, and $v_{measured}$, and the average velocities over various time intervals. We don't have the final instantaneous velocities at all the gate positions because we did not measure all the Δ**Time 3**'s. We can, however, derive an equation relating $v_{average}$, v_r, a and **t**. (*The* v_0 *in the velocity equation is the instantaneous velocity when the clock reads zero, while* v_r *is the instantaneous velocity when* Δt *is zero. Don't confuse them.*)

It can be shown, see handout, that

$$v_{average} = v_r + \tfrac{1}{2}\,a\,\Delta t$$

This is a straight-line relationship, just like the velocity equation, except that the slope of this seldom-used equation is equal to one-half of the acceleration. Test it to see if this is true.

a (Graph #1) = ____________ m/s^2 twice the slope = 2**m** = a (Graph #2) = ____________ m/s^2

Do the two graphs agree on their estimate of acceleration? _________, %Diff = ________________%
Use **a** from Graph #1 as reference

If the predicted accelerations are in good agreement, then this average velocity relationship is confirmed.

16. Now compare the y-intercept of the Average Velocity vs Time Interval graph with your three other estimates of the instantaneous velocity at $\boldsymbol{x}$ = 0.800 m. You will discuss this below. Table III contains three estimates of the instantaneous velocity, $\boldsymbol{v_r}$, at ($\mathbf{t}_r$, $\boldsymbol{x}_r$ = 0.800 m). The y-intercept on Graph #2 provides you with yet another estimate of $\boldsymbol{v_r}$ at ($\mathbf{t}_r$, $\boldsymbol{x}_r$ = 0.800 m).

$$\mathbf{b} = \boldsymbol{v}_{\boldsymbol{Graph\ \#2}} = \_\_\_\_\_\_\_\_\_\_\_\_\ \text{m/s}$$

Use the space below and on the next page to compare your four independent estimates of the instantaneous velocity, $\boldsymbol{v_r}$, at ($\mathbf{t}_r$, $\boldsymbol{x}_r$ = 0.800 m), which you have obtained from these exercises. Think beyond your own personal results and describe what general conclusions about instantaneous velocity and its measurement you have learned through these exercises. Write neatly or type in a word processor and paste your essay into the spaces provided below and on the next page. (*The discussion should be both qualitative and quantitative*.)