

Velocity Equation for Average Velocity vs Time Interval; $v_{average}$ vs Δt

Derivation of the equation for $v_{average}$ as a function of Δt (a and v_r both constant)

In this derivation Δt is always a time interval that starts at a reference time $\Delta t = t - t_r$. The time interval can be positive or negative, depending on whether you are looking back in time or forward in time. Time intervals to times earlier than t_r are negative. Time intervals to times later than t_r are positive.

All the velocities in this derivation, except $v_{average}$, are instantaneous velocities. Working with Δt can be a little tricky, especially when the position at which $\Delta t = 0$ is in the middle of the data set; as it is in Lab 05. -- So pay close attention to this derivation.

This derivation will show that the following equation describes the relationship between the instantaneous velocity at $\Delta t = 0$, v_r , and the average velocity, $v_{average}$, over a time interval, Δt , at constant acceleration.

$$v_{average} = v_r + \frac{1}{2} a \Delta t$$

In this derivation, we will use one of the equations we discovered in an earlier lab, namely

$$v_{average} = (v_r + v) / 2$$

where v is the instantaneous velocity at time t , and v_r is the instantaneous velocity at time the reference time, t_r , or what amounts to the same thing, when $\Delta t = 0$. Thus, the algebra proceeds as follows,

$$2 v_{average} = (v_r + v)$$

Now, rearrange and note that v is given by the velocity equation, yielding

$$2 v_{average} - v_r = v = v_r + a \Delta t$$

The first and last expressions yield

$$2 v_{average} = 2 v_r + a \Delta t$$

and, finally yield

$$v_{average} = v_r + \frac{1}{2} a \Delta t$$

When the acceleration is constant, we expect that a plot of average velocity vs time interval to be a straight line with a y-intercept equal to the instantaneous velocity, v_r , at $\Delta t = 0$, with a slope equal to one-half the acceleration, $\frac{1}{2} a$.

Don't forget that we can calculate average velocity directly from the displacement data.

$$v_{average} = (x - x_r) / \Delta t$$

Like Δt , the displacement in the numerator can be positive or negative, depending on whether you are looking at positions from earlier times or at positions from later times.