Name: ______________________________ **Period:** _____ **Due Date:** ____________
Lab Partners: ____ **Station Letter:** ___

ROTATIONAL EQUILIBRIUM – WebAssign

Theory: An object at equilibrium must have zero acceleration in both its linear and rotational motions. Thus its linear velocity, **v**, and its angular velocity, **ω**, must both be zero or constant. When an object is at equilibrium, all the forces acting on it cancel out. We say, the sum of the external forces is zero;

$$\Sigma \mathbf{F}_{EXT} = \mathbf{0}$$

Furthermore, when an object is at equilibrium, all the torques acting on it must also cancel out. We say the sum of the torques is zero;

$$\Sigma \tau = \mathbf{0}$$

You can easily verify by eye that a system is at equilibrium in the special case where it's center of mass is not moving, **v** = 0 = constant, and where the object is not rotating, **ω** = 0 = constant. We will examine this special equilibrium case today.

In this laboratory exercise you will investigate these two mathematical criteria for equilibrium by working with a system subjected to a number of forces while at equilibrium. You will be asked to solve for two "unknown" forces, the tension in a bungee, **T**, and the weight of the meter stick, $\mathbf{F}_M$.

Finding two unknowns always requires that we solve two equations simultaneously. Two or more simultaneous equations is known as a system of equations. In this case, there are two equations. They are, $\Sigma \mathbf{F}_{EXT} = \mathbf{0}$ and $\Sigma \tau = \mathbf{0}$. Solving these two equations for the two unknowns is simplest if one of the two equations contains only a single unknown. By assuming a point of rotation at the attachment point of the bungee, the tension in the bungee disappears from the torque equation. That allows us to find the weight of the meter stick using the torque equation alone. The force equation can then be solved for the only remaining unknown force, the tension in the bungee. The true weight of the meter stick will be determined by another method at the end of the lab. At that time, you will compare the true mass of the meter stick with your experimental estimates.

It is expected that you will verify the equilibrium equations by confirming that the sum of the torques and the sum of the forces equal zero. To some extent your lab grade depends on how well you are able to measure the forces and torques to confirm these equalities. Care and attention to detail will increase your accuracy and, to a small degree, that care will be reflected in your grade.

General Procedure

A meter stick, with its attached hardware, is subjected to four external forces. Two are upward forces supplied by a bungee, **T**, and a spring, $\mathbf{F_S}$. The third and fourth forces are supplied by the weight of a hanging mass, $\mathbf{W_W}$, and the weight of the meter stick itself, $\mathbf{W_M}$. Both gravity-based forces point straight down. You can change the torques exerted on the meter stick by changing the position of the hanging mass. In the final section, you will change the angle of the two upward forces to verify that only the vertical components are important when computing the torque. The horizontal components must cancel each other out,otherwise the meter stick would be moving horizontally.

In all that follows, the weight of the meter stick, including its three attached hangers, and the tension in the bungee are true unknowns. You will use the equilibrium equations for force and torque to solve for these two unknown forces in each part of the lab. The tension is unique to each part. The mass of the meter stick, however, is constant and we can compare its value, as determined by each part, to the true mass, which you will determine later.

The mass of the hanging weight is known. Record that information here and calculate its weight.

Mass of hanging weight(s) = __________________ grams = ________________ kg

$\mathbf{F_W}$ = Weight of hanging weight(s) = mass in kg x 9.81 N/kg = ______________ N

The weight of the meter stick and hangers will be treated as though it is located at the center of the meter stick.

We assume the center of mass of the meter stick and hangers remains at the 50 cm mark at all times.

This assumption introduces a negligible error into your results in Parts II , III, and IV.

Begin by verifying that the apparatus is arranged with the parts in the specified positions in each section. The bungee and spring must both be **vertical** for all measurements made in Parts I, II, and III, but not in Part IV. The two vertical support rods must remain 70 cm apart and the meter stick must be **horizontal** at the time when you read the spring scale.

In our calculations, all forces are positive. Positive and negative signs appear in the equations to indicate direction, + is upward and – is downward. Similarly, in the torque equation, all torques are positive numbers. The equations include signs for the directions; + for CW and – for CCW rotation.

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Suggestions for writing force and torque equations: First, use symbols rather than numbers to write all equations. Then substitute known values for the symbols. This will greatly simplify the ease of interpreting the equations. Use distinct symbols for each force or component. Here are the symbols we'll be using in this lab: (*Make sure you use them, too.*)

For Parts I, II and III		**For Part IV only .**
$\mathbf{F_M}$ = weight of meter stick & hangers	**T** = tension in the bungee	$\mathbf{T_i}$ & $\mathbf{T_j}$ = **T** cos ϕ & **T** sin ϕ
$\mathbf{F_W}$ = weight of hanging weight	$\mathbf{F_S}$ = force from the spring	$\mathbf{F_i}$ & $\mathbf{F_j}$ = $\mathbf{F_S}$ cos θ & $\mathbf{F_S}$ sin θ

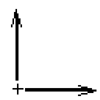
Part I

Bungee at 15 cm mark   Hanging mass at 50 cm mark   Spring at 85 cm mark

Record the spring reading off the grams scale and calculate the spring force, $\mathbf{F_S}$, in newtons.

Spring: ___________ g $\Rightarrow$ $\mathbf{F_S}$ = ____________ kg x 9.81 N/kg = ___________ N

Draw the free-body diagram of the forces acting on the meter stick. (*Show all four forces.*)



(*Here are the two equations you must solve. They are written out for you here. Write these equations for yourself in Parts II and III.*)

Assume a pivot point (**X**) at the place where the bungee is attached. Begin by writing an equation showing that the sum of the torques about this pivot is zero. It will contain one unknown force. **Circle it**. (*Be sure you change the distances to meters in the torque calculations on Parts II, III and IV. It was done for you here.*)

$\Sigma\, \boldsymbol{\tau} = +\mathbf{F_M}\bullet(0.350) + \mathbf{F_W}\bullet(0.350) - \mathbf{F_S}\bullet(0.700) = +\,\mathbf{F_M}\bullet(0.350) +$ ____________ $-$ ____________ $= \mathbf{0}$

Fill-in the known torques here.

Continue by writing an equation showing that the sum of the upward and downward forces is zero. This equation has two unknowns. **Circle them**.

$\Sigma\, \mathbf{F_{EXT}} = +\mathbf{T} + \mathbf{F_S} - \mathbf{F_M} - \mathbf{F_W} = +\ \mathbf{T}\ +$ ____________ $-\ \mathbf{F_M}\ -$ ____________ $= \mathbf{0}$

Solve the first equation for the weight of the meter stick, $\mathbf{F_M}$. Then use that result along with the second equation to solve for the tension in the bungee, **T**.

Weight of meter stick = $\mathbf{F_M}$ = _____________ N

Tension in the bungee = **T** = _____________ N

Now, calculate the mass of the meter stick.

Mass of the meter stick = $\mathbf{F_M}$ / (9.81 N/kg) = ___________ kg = ___________ g

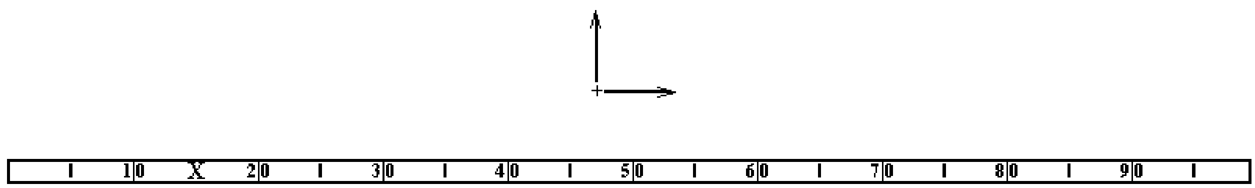
Part II

Bungee at 15 cm mark   Hanging mass at 40 cm mark   Spring at 85 cm mark

Record the spring reading off the grams scale and calculate the spring force, $\mathbf{F_S}$, in newtons.

Spring: ___________ g $\Rightarrow \mathbf{F_S}$ = ____________ kg x 9.81 N/kg = ___________ N

Draw the free-body diagram of the <u>forces</u> acting on the meter stick. (*Show all four forces.*)



Assume a pivot point (**X**) at the place where the bungee is attached. Begin by writing an equation showing that the sum of the torques about this pivot is zero. It will contain one unknown force. **<u>Circle it</u>**.

$$\Sigma \boldsymbol{\tau} = +\mathbf{F_M}\bullet(0.350) + \mathbf{F_W}\bullet(\_\_\_\_\_\_) - \mathbf{F_S}\bullet(\_\_\_\_\_\_) = +\mathbf{F_M}\bullet(0.350) + \_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_ = \mathbf{0}$$

Fill-in the known torques here.

Continue by writing an equation showing that the sum of the upward and downward <u>forces</u> is zero. This equation has two unknowns. **<u>Circle them</u>**.

$$\Sigma \mathbf{F_{EXT}} = +\mathbf{T} + \mathbf{F_S} - \mathbf{F_M} - \mathbf{F_W} = +\ \mathbf{T}\ + \_\_\_\_\_\_\_\_\_ - \ \mathbf{F_M}\ - \_\_\_\_\_\_\_\_\_ = \mathbf{0}$$

Solve the first equation for the weight of the meter stick, $\mathbf{F_M}$. Then use that result along with the second equation to solve for the tension in the bungee, **T**.

Weight of meter stick = $\mathbf{F_M}$ = _____________ N

Tension in the bungee = **T** = _____________ N

Now, calculate the mass of the meter stick.

Mass of the meter stick = $\mathbf{F_M}$ / (9.81 N/kg) = ___________ kg = ___________ g

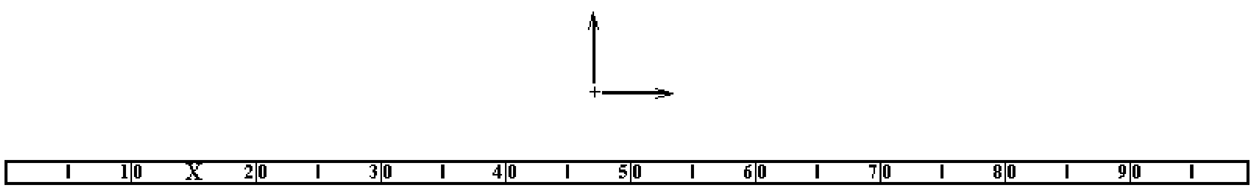
Part III

Bungee at 15 cm mark **Hanging mass at 60 cm mark** **Spring at 85 cm mark**

Record the spring reading off the grams scale and calculate the spring force, $\mathbf{F_S}$, in newtons.

Spring: ____________ g $\Rightarrow$ $\mathbf{F_S}$ = _____________ kg x 9.81 N/kg = ____________ N

Draw the free-body diagram of the <u>forces</u> acting on the meter stick. (*Show all four forces.*)



Assume a pivot point (**X**) at the place where the bungee is attached. Begin by writing an equation showing that the sum of the torques about this pivot is zero. It will contain one unknown force. **<u>Circle it</u>**.

$\Sigma\ \boldsymbol{\tau} = +\mathbf{F_M}\bullet(0.350) + \mathbf{F_W}\bullet(\_\_\_\_\_\_\_) - \mathbf{F_S}\bullet(\_\_\_\_\_\_\_) = +\ \mathbf{F_M}\bullet(0.350) + \_\_\_\_\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_\_\_\_\_ = \mathbf{0}$

Fill-in the known torques here.

Continue by writing an equation showing that the sum of the upward and downward <u>forces</u> is zero. This equation has two unknowns. **<u>Circle them</u>**.

$\Sigma\ \mathbf{F_{EXT}} = +\mathbf{T} + \mathbf{F_S} - \mathbf{F_M} - \mathbf{F_W} = +\ \mathbf{T}\ + \_\_\_\_\_\_\_\_\_\_\_\_ -\ \mathbf{F_M}\ - \_\_\_\_\_\_\_\_\_\_\_\_ = \mathbf{0}$

Solve the first equation for the weight of the meter stick, $\mathbf{F_M}$. Then use that result along with the second equation to solve for the tension in the bungee, **T**.

Weight of meter stick = $\mathbf{F_M}$ = _____________ N

Tension in the bungee = **T** = _____________ N

Now, calculate the mass of the meter stick.

Mass of the meter stick = $\mathbf{F_M}$ / (9.81 N/kg) = ___________ kg = ___________ g

Part IV (*This one is different from the first three, so read it carefully*)

Bungee at 0 cm mark **Hanging mass at 60 cm mark** **Spring at 100 cm mark**

Record the spring reading off the grams scale and calculate the spring force, $\mathbf{F_S}$, in newtons.

Spring: ___________ g ⇒ $\mathbf{F_S}$ = ____________ kg x 9.81 N/kg = ___________ N

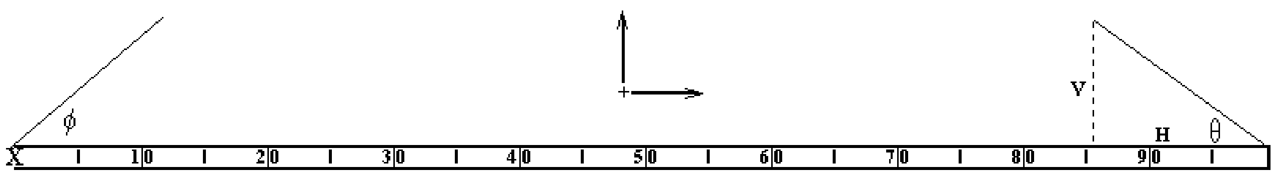
Measure the angle the spring makes with the meter stick:

horizontal (î) distance = H = _________ cm; vertical (ĵ) distance = V = __________ cm.

Use the arctan ($\tan^{-1}$) function to calculate the angle of the spring force.

$\tan^{-1}(V / H) = \theta$ = ___________° = angle of spring force vector.

Draw the free-body diagram of all the forces. (*There are four forces. Show both components of non-vertical forces.*)



A. Calculate the components of the spring force: $\mathbf{F_S} = \mathbf{F_i}\,\hat{\imath} + \mathbf{F_j}\,\hat{\jmath}$ = – ____________ î + ___________ ĵ **N**
($\mathbf{F_i}$) ($\mathbf{F_j}$)

B. Next, solve the Sum of Torques equation for the weight of the meter stick, $\mathbf{F_M}$. Then calculate its mass. Assume a pivot point (**X**) at the place where the bungee is attached. Write the equation showing that the sum of the torques about this pivot point is zero. It contains one unknown force. **Circle it**.

$\Sigma\,\tau = +\mathbf{F_M}\bullet 0.500 + \mathbf{F_W}\bullet 0.600 - \mathbf{F_j}\bullet 1.000 = +\,\mathbf{F_M}\bullet 0.500 +$ __________ – __________ = **0**

Fill-in the known torques here.

Therefore, $\mathbf{F_M}$ = _________ N

Mass of the meter stick = $\mathbf{F_M}$ / (9.81 N/kg) = ____________ kg = __________ g

C. Now, write an equation showing that the Sum of the Horizontal Force Components (î) equals zero. There are only two: the horizontal component of the spring force and the horizontal component of the tension. Solve this equation for the **î**-component of the tension in the bungee.

$\Sigma\,\mathbf{F_H} = \mathbf{T_i} - \mathbf{F_i} = \mathbf{T_i}$ – ____________ = **0** ; Therefore, $\mathbf{T_i} = \mathbf{F_i}$ = _________ N
($\mathbf{F_i}$)

D. Then, write an equation showing that the Sum of the Vertical Force Components (ĵ) equals zero. Use the result from **B.** to solve it for the **ĵ**-component of the tension, T_j:

$\Sigma\,\mathbf{F_V} = \mathbf{T_j} + \mathbf{F_j} - \mathbf{F_M} - \mathbf{F_W} = \mathbf{T_j}$ + ________ – $\mathbf{F_M}$ – _________ = **0**; Therefore, $\mathbf{T_j}$ = _________ N

E. Finally, write the bungee tension vector in both component ($T_i\,\hat{\imath} + T_j\,\hat{\jmath}$) and polar ($T \angle \phi$) forms:

Bungee tension = **T** = ______________ N î + ______________ N ĵ = ______________ N ∠ _________°
($\mathbf{T_i}$) ($\mathbf{T_j}$) (**T**) (ϕ)

Name: ___________________________________ Period: _____ Due Date: ___________

Comparing your experimental estimate of the mass of the meter stick to its true mass.

List your results for the mass of the meter stick from each of the four parts of this lab.

	Mass
Part I	___________ g
Part II	___________ g
Part III	___________ g
Part IV	___________ g

We will use an analytical balance to determine the true mass of your meter stick.

True mass of meter stick = ___________ g

Calculate the percent error in the estimated mass of the meter stick in each part of the lab.

	%Error
Part I	___________ %
Part II	___________ %
Part III	___________ %
Part IV	___________ %