

Name: _____ Period: _____ Due Date: _____
 Lab Partners: _____

CALCULATING THE FORCE AND IMPULSE OF A COLLISION - WebAssign

Purpose: Use the impulse-momentum equation to analyze the dynamics of a simple collision. Conceptually, this lab is divided into two parts. In Part A, we make measurements that allow us to calculate the velocity of the glider at the instant the collision begins. The final velocity of Part A then becomes the initial velocity of the collision examined in Part B. The meaning of the terms “initial velocity” and “final velocity” depend on which part of the lab you are discussing. Consider carefully whenever you are asked to specify one of these velocities; to which part of the lab the question is referring?

Theory: If an external force (F) acts on an object over an interval of time (Δt), we call the product $F\Delta t$ the *impulse*. According to Newton's Second Law,

$$F = ma,$$

and, because $a = \Delta v / \Delta t$, for small, time intervals, where $v_{\text{ave}} = v_{\text{inst}}$, we can rewrite this equation as

$$F = m (\Delta v / \Delta t)$$

from which we derive the relationship
 between **impulse** and **change in momentum**

$$F\Delta t = m\Delta v = m(v_f - v_i)$$

We call mv (mass • velocity) the *momentum* of an object, and we call $m\Delta v$, or $\Delta(mv)$ in those occasional situations where the mass also varies, its **change in momentum**. Thus, we can restate the final equation, in words, as follows:

The impulse experienced by an object during a collision equals its change in momentum.

This is a classic statement of the **Impulse-Momentum Equation**.

We can measure the impulse experienced by an object during a collision by measuring either pair of parameters, F and Δt , or m and Δv . Today we will try something a little more ambitious. We'll measure the change in momentum ($m\Delta v$) and the duration (Δt) of the collision. Then we will measure Δt and then use the impulse-momentum relationship to calculate the average force acting on the object during the collision.

Finding the average force is equivalent to assuming the force is constant. We know as a matter of simple logic that the force is not constant. Averaging gives us only general information about the collision. Detailed moment-to-moment velocities of the object during the collision are required for us to know the instantaneous force acting on the object at each moment during the collision. The average force vs. Δt graph has the same area as the instantaneous force vs. Δt graph. Both equal the change in momentum. We will not try to construct the detailed force vs. time graph.

Part A: Obtaining Velocity vs. Time data prior to the collision (Note: acceleration of the system is constant.)

1. Elevate one end of an air track about one inch. Place a rubber-band bumper at the other end.
2. Place photogate #1 at the 30 cm point on the track.
3. Place photogate #2 at the 80 cm point on the track.
4. Open up *Logger Pro* on the computer and open the V_avg and A_avg experiment file.
5. Obtain a glider and place a post-it note at the **front end**.
6. Release the glider from the top of the elevated end of the track and record times until you can satisfactorily reproduce the times.
7. Record the three times in Data Table I
8. Move photogate #2 to positions of 110 cm, 140 cm, and 170 cm, and repeat steps 6-7.

DATA TABLE I – POSITION VS. TIME INFORMATION FOR THE GLIDER

Initial Position	Final Position	Time 1	Time 2	Time 3
0.30 m	0.80 m	_____	_____	_____
0.30 m	1.10 m	_____	_____	_____
0.30 m	1.40 m	_____	_____	_____
0.30 m	1.70 m	_____	_____	_____

(It's not the fall, but the sudden stop, that really stings. As you proceed, imagine how your analysis helps to explain this truism. Can you explain why automobile manufacturers advertise their "crumple zones" as a car safety feature that protects passengers during collisions? OR How stunt-performers can jump off a building and survive?)

Analysis Part A: Determining the Initial Velocity of the Glider before the Collision

(This must be finished before the end of the lab period, including a preliminary version of Graph 1)

In order to determine the average force of the collision between the rubber band and the glider, we need the initial and final momenta of the glider. Since momentum $p = \text{mass} \cdot \text{velocity}$, we need to know the velocity of the glider just before and just after the collision. Remember, the final velocity at the end of the run down the rail becomes the initial velocity just before the collision. We need the velocity just before the collision in order to analyze the collision.

A1. We wish to construct a position vs. time graph for the glider as it travels down the track. To do this add Time 1 and Time 2 from each of the entries in Data Table I and record this information in Table II. This is the elapsed time for the glider as it passed between the positions of the two photogates.

TABLE II – Position vs. Elapsed Time of the Glider (Before the collision)

Position	Elapsed Time = t (Time 1 + Time 2; From Data Table I)
0.3000 m	_____ 0.000 sec This is time t = 0 s on the graph
0.8000 m	_____
1.1000 m	_____
1.4000 m	_____
1.7000 m	_____

A2. Using **Graphical Analysis**, construct **Graph #1** of **Position** (of photogate #2) vs. **Elapsed Time** using the information in Table II. Give your Elapsed Time column the short name **t**. Don't forget to include the starting data point: (**t** = 0.000 s, **x₀** = 0.3000 m). Set the maximum value on the vertical axis to +2.10 m and the minimum vertical value to -0.5 m. The minimum value on the horizontal time axis must be set to -1.5 s and the maximum value on the time axis should be set to +4.0 s.

A3. Using the curve fit option in *Graphical Analysis*, fit your data to **DEFINE** a quadratic function of the form

$$y = 0.30 + B*t + 0.5*A*t^2.$$

Now, write the specific equation, with numerical coefficients, for glider position vs. time on the line below.

Position vs. Elapsed Time for the glider is $x = 0.30 + Bt + \frac{1}{2}At^2 = 0.30 + \underline{\hspace{2cm}} t + \frac{1}{2} \underline{\hspace{2cm}} t^2$

A4. Place the glider on the track and note the position where it first touches the rubber band = m

A5. Use the **interpolate** feature of **Graphical Analysis** on Graph #1 to estimate the time corresponding to the position of the glider as it is about to touch the rubber band (this position should also be recorded in Data Table III). Record the estimate of the time required to reach this position. (*Interpolate cannot give the exact time or position so this feature only gives you a rough estimate. Just get as close as you can. You want this rough estimate primarily to satisfy yourself that the calculated result in the next paragraph is correct.*)

Time estimate for the Glider to reach the rubber band = s (interpolated on Graph #1)

A6. Now, use your position equation from Part A3. Set **x** equal to the position of the glider as it is about to touch the rubber band to solve for the time it took the glider to reach that position. This should be the most accurate way to calculate the time before collision and should be reasonably close to the estimate above using **Graphical Analysis'** interpolate feature. (*Using the quadratic formula is one method of solving for the time. The TI-83 has a solver feature that can help you with that chore.*)

Time estimate for the Glider to reach the rubber band = s (Solve the position equation for t)

(*The two time estimates, A5 and A6, should agree with each other. For 1 inch of elevation, the time should be close to 2.5 seconds.*)

A7. Your position vs elapsed time equation in step A3 above describes the motion of the glider **for any time up to the moment of collision**. How can we convert this to velocity information? By using the **derivative**, of course! Take the derivative of your position vs elapsed time equation and find the velocity equation for the glider. Substitute the time from section A6 to find the velocity of the glider as it is about to touch the rubber band. This is the velocity of the glider just before the collision. (***You already have the position of the glider just before the collision, the time it takes the glider to get to the collision point, and now you are ready to calculate the instantaneous velocity of the glider at the moment before the collision.***) Show your work here (especially, taking the correct derivative) and enter the answer on the line below.

Instantaneous Velocity of Glider as it reaches the rubber band just before the Collision: m/s

(*When using a small block this velocity should be close to but probably less than 1.00 m/s*)

(Hint: the derivative of the position equation is the velocity equation.)

Procedure Part B: Determining the Duration and Final Velocity of Three Collisions

Before you begin, adjust the rubber band so that you obtain a “sweet” collision with and without the weights on the glider. This is like hitting a golf ball or a baseball in the sweet spot on the club or the bat. You’ll get a smooth and soundless collision from the glider. It is important to get a nearly perfect collision. The glider should bounce 90% of the way back to its starting point. When you loosen the rubber band for the third collision it should be only slightly loosened. For the second collision with the heavier glider the rubber band should work without any adjustment. In all cases, make sure the glider does not hit the back of the fork holding the rubber band. A finger on the **end-stop** is the best way to detect such a flawed collision. All collisions should be soundless - no noise from the glider or the track.

1. Determine the mass of the glider using the scale.
2. Position photogate #1 so it is blocked (i.e. that the post-it interrupts the beam) just as the glider makes first contact with the rubber band.
3. Position photogate #2 so the post-it note on the glider enters photogate #2 on the rebound just after it leaves photogate #1. (The time delay between leaving photogate #1 and entering photogate #2 should be very short for optimum performance – Typically, Time #2 about 5 milliseconds = 0.005 s.)
4. Record the position of the glider (the front edge i.e. down-hill edge of the glider) when photogate #1 is about to be blocked at the beginning of the collision. (The position should be close to **1.97 meters**.)
5. Release the glider from the top of the track. The glider will hit the rubber band and bounce back up the track. Catch the glider once it has passed through both photogates following the collision.
6. Each collision is unique. There is no need for multiple trials. Record your timing data in Data Table III.
7. Repeat the experiment by changing the mass of the glider, then by loosening the rubber band (*only slightly*).

DATA TABLE III – DURATION OF COLLISION AND FINAL VELOCITY OF THE GLIDER

Mass of Glider: _____ g = _____ kg

Length of Post-it Note: _____ mm = _____ m

Position of Glider as it reaches the rubber band just before the collision (see A4. above): _____ m

Trial #	Time 1	Time 2 (Not used)	Time 3	
1	_____	_____	_____	(glider w/no weights + tight rubber band)
2	_____	_____	_____	(glider w/100 grams + tight rubber band)
3	_____	_____	_____	(glider w/no weights + loosened rubber band)

We can ignore Time 2 because it only measures the transition time between the two photogates. We ignore it except to note that it must be small so that we are sure to measure the final velocity as quickly as possible after the collision.

If your Time 2 times are not very short, on the order of a few milliseconds, check immediately with your instructor. This data must to be correct before you try to continue with the analysis.

Analysis Part B: Finding the Average Force and Impulse of the Collision

B1. All collisions have the same initial velocity because the glider always comes the same distance down the rail under the influence of the same constant force – gravity. Gravity imparts a constant acceleration independent of the mass of the glider as long as the tilt is constant. The final velocity of the run down the rail is the initial velocity of the collision.

In order to calculate the average force between the rubber band and the glider, we need the momentum of the glider both before and after the collision as well as the duration of the collision. In Part A we calculated the velocity of the glider just before the collision. From this information and the mass data in Table III you can determine the initial momentum of the glider just before the collision.

Even though all collisions have the same initial velocity, they do not all have the same initial momentum. Why not?

B2. Enter the initial velocity of the glider calculated in Part A7, then calculate the initial momentum of each collision.

Trial #	Velocity of glider before the collision	Momentum of the glider: before the collision
1	$v_{i1} = \underline{\hspace{2cm}} \text{ m/s}$	$m_1 \cdot v_{i1} = \underline{\hspace{2cm}} \text{ kg m/s (glider + tight rubber band)}$
2	$v_{i2} = \underline{\hspace{2cm}} \text{ m/s}$	$m_2 \cdot v_{i2} = \underline{\hspace{2cm}} \text{ kg m/s (glider w/100 grams added)}$
3	$v_{i3} = \underline{\hspace{2cm}} \text{ m/s}$	$m_3 \cdot v_{i3} = \underline{\hspace{2cm}} \text{ kg m/s (glider + loosened band)}$

B3. Time 3 from Data Table III gives the information necessary to calculate the final velocity of the glider. Recall that Time 3 is the time it takes the post-it to pass through photogate #2 after the collision. We only need to divide the post-it note length by Time 3 to obtain the final velocity and final momentum of the glider after each collision. ***(Note that the final velocity must be negative because the Glider is moving in the opposite direction.)*** Use the final velocity to calculate the final momentum of the glider after each collision. ***The final momentum must also be negative because it is based on the final velocity, which is negative.*** Do these calculations for all three collision trials.

Trial #	Velocity of the glider after the collision:	Momentum of glider after the collision
1	$v_{f1} = \underline{\hspace{2cm}} \text{ m/s}$	$m_1 \cdot v_{f1} = \underline{\hspace{2cm}} \text{ kg m/s (glider + tight rubber band)}$
2	$v_{f2} = \underline{\hspace{2cm}} \text{ m/s}$	$m_2 \cdot v_{f2} = \underline{\hspace{2cm}} \text{ kg m/s (glider w/100 grams added)}$
3	$v_{f3} = \underline{\hspace{2cm}} \text{ m/s}$	$m_3 \cdot v_{f3} = \underline{\hspace{2cm}} \text{ kg m/s (glider + loosened rubber band)}$

B4. Now **use all you have calculated above** to complete the table below for three collisions. The first four columns simply collect earlier calculations. The last three columns are calculated from the first four columns.

1. Transfer the initial and final velocities and the initial and final momenta to Table IV. (show 4 digit precision.)
2. Find the change in momentum by subtracting (final momentum – initial momentum). The result will be a negative number. (Be sure that you treat the positive and negative signs correctly.)
3. Record the duration of the collision (Δt = **Time 1 from Table III**) for each trial.
4. The average force is then calculated using the impulse- momentum equation.

TABLE IV – AVERAGE FORCE OF COLLISION BETWEEN THE GLIDER AND THE BAND

We use the following symbols: v_i and v_f for the initial and final velocities of the collision, m for the mass of the glider and Δt for the duration of the collision. F_{ave} is the average force of the rubber band on the glider during the collision.

When calculating the momentum difference remember that the final momentum is already negative.

For example, if the initial momentum is 2 kg m/s and the final momentum is -1.5 kg m/s, then the momentum difference will be $mv_f - mv_i = -1.5 - 2 = -3.5$ kg m/s. As a result, the average force will also be negative. Negative simply means that the change in momentum and the average force both point back up the incline when we selected the positive direction to point down the incline.

Trial#	v_f	v_i	mv_f	mv_i	$mv_f - mv_i$	$\Delta t = T1$	F_{ave}
1. (glider + tight rubber band)	_____	_____	_____	_____	_____	_____	_____
2. (glider w/100g + tight rubber band)	_____	_____	_____	_____	_____	_____	_____
3. (glider + loosened rubber band)	_____	_____	_____	_____	_____	_____	_____

B5. The initial velocities are the same for all three collisions; why do the average forces differ?

B6. The glider loses its positive momentum because of the force of the rubber band, where does that momentum go?
