Name: ______________________ **Period:** _____ **Due Date:** ___________
Lab Partners: ____

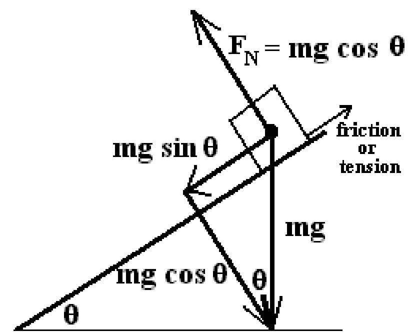
THE PHYSICS OF INCLINES - WebAssign

Purpose: To study the physics of inclines with and without friction.

Theory: When an object is on an incline its weight has two components (parallel and perpendicular to the incline):

1) **Its weight pulls the object along (parallel to) the incline, and**

2) **Its weight pulls the object onto (perpendicular to) the incline.**



These two components are perpendicular to each other. One component of the weight is parallel to the surface of the incline and tries to accelerate the object down the incline. The other component of the weight is perpendicular to the plane of the incline and it can effect the motion of the object only through its effect on the normal force and thus on the force of friction. The object's weight and the angle of tilt are used to calculate both components of the weight.

If the angle of inclination (AKA elevation angle or inclination angle) is θ, then the component of the gravitational force along (or parallel to) the incline is **mg sin θ.** The component of the gravitational force into (or perpendicular to) the incline is **mg cos θ**. Acceleration is parallel to the incline, not perpendicular to it. The normal force is equal to but points opposite to **mg cos θ**. There are no other external forces on the object, only the weight and the normal force. The coordinate system we use has one axis parallel (‖) to the incline and one axis perpendicular (⊥) to the incline.

$$\Sigma F_{\perp} = F_N - mg\cos\theta = 0$$
therefore,
$$F_N = mg\cos\theta$$

The net force and the acceleration will always be parallel to the plane and **if** there is no friction

$$F_{NET} = \Sigma F_{\parallel} = mg\sin\theta = ma$$
therefore,
$$a = g\sin\theta \qquad \textit{Equation 1}$$

This equation describes the situation you will encounter in **Part One** of the lab where the cart slides frictionlessly down the slope of the tilted air track. The acceleration increases as the angle of inclination increases. When the angle is zero, the acceleration is zero. When the angle is 90º, the acceleration equals 9.81 m/s^2, since **sin 90º** = 1. Between 0º and 90º the acceleration varies between 0.00 m/s^2 and 9.81 m/s^2.

By adjusting the angle of inclination we can tune the acceleration to any value between 0.00 m/s^2 and 9.81 m/s^2.

You will investigate only a narrow range of relatively small angles in **Part One**.

In **Part Two,** you explore what happens when an object sits on an incline and friction is present. In particular, we will look at the equilibrium situation where the friction force is large enough to stop the object from moving down the incline.

If <u>friction is present</u>, then the parallel force equation is modified by the inclusion of the friction force to yield

$$\mathbf{F_{NET} = \Sigma F_{\|} = mg\ sin\ \theta - \mu_k\ F_N = mg\ sin\ \theta - \mu_k\ mg\ cos\ \theta = ma}$$

therefore,

$$\mathbf{a = g\ sin\ \theta - \mu_k\ g\ cos\ \theta = g\ \{\ sin\ \theta - \mu_k\ cos\ \theta\ \}}$$

If friction is present and **if** the object on the plane is not moving, **then a** = 0, the coefficient of kinetic friction is replaced by the coefficient of static friction in the above equation, and we find that

$$\mathbf{g\ sin\ \theta = \mu_S\ g\ cos\ \theta}$$

therefore,

$$\mathbf{sin\ \theta = \mu_S\ cos\ \theta}$$

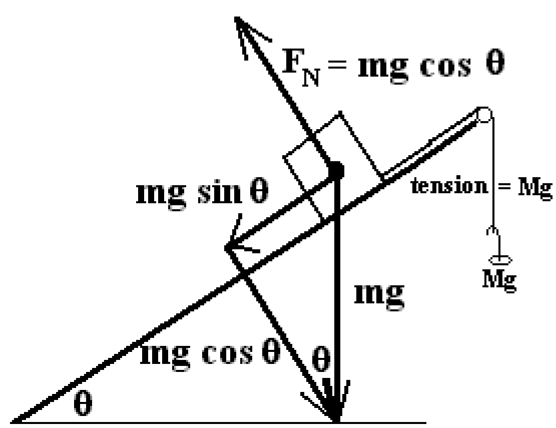
therefore,

$$\mathbf{\mu_S = sin\ \theta / cos\ \theta = tan\ \theta} \qquad \textit{Equation 2}$$

To find the true value of the coefficient of static friction (μ_S) in *Equation 2*, you need to adjust the angle of inclination until the force along the incline is large enough to produce the maximum possible static friction force. Recall the μ_S is only used to calculate the largest possible value of the static friction force. Therefore, we need to adjust the static friction force to its maximum value in order to find the true value of μ_S.

You will investigate the static friction force using a variety of objects on an incline with friction. You will determine by measurement and calculation the coefficient of friction between each object and the surface of the incline.

In **Part Three,** you will investigate a case where another, non-friction, force keeps the object from moving down the incline. This is another equilibrium case. From the diagram on page 1, it should be clear that a string pulling up the hill can be just as effective as friction in preventing the object from moving down the incline.



We will use a cart with minimal friction for this part of the lab. A string passing over a pulley will attach it to a hanging weight. As we vary the hanging weight, you will determine the angle, θ, at which the tension in the string is equal and opposite to the component of the weight pulling the cart down the slope. The diagram on the right shows the setup.

We can vary the tension in the string by adding weight to the hanger on the end of the string. For each tension, there will be a unique value for θ. The analysis of the situation depends on your understanding that at equilibrium the component of the weight pointing down the slope (**mg sin** θ) is equal in magnitude to the tension in the string pointing up the slope. Furthermore, the tension in the string is equal to the total weight of the hanger (**Mg**). Therefore,

$$\mathbf{mg\ sin\ \theta = Mg}$$

or

$$\mathbf{m\ sin\ \theta = M} \qquad \textit{Equation 3}$$

A graph of **M** vs **sin** θ will be a straight line with a zero intercept and a slope equal to the mass of the cart (**m**).

Part One: Acceleration on a Frictionless Incline

1. Attach a flag to an air track glider. Measure the length of the flag and record it in Data Table I.
2. Level the air track. Then elevate the track and record the height of the elevating block.
3. Place two photogates so that the flag will pass through them one at a time. You may put the photogates almost anywhere on the track: the distance between them will not matter in these calculations.
4. Start LoggerPro and open the file Vave_and_Aave. You will collect three columns of time data labeled T_1, T_2 and T_3. (This should be very familiar by now.)
5. Let the glider slide from the top end down the incline and record the times.
6. Repeat the procedure at a total of five different elevations. Record all the data in Data Table I.

Data Table I

Width of flag = ______________ cm = ______________ meters

Height of 10 identical thin books = _____________ meters; Height of one book = _____________ meters

# of Books	Track Elevation (m)	Time #1 (s)	Time #2 (s)	Time #3 (s)
____	________________	_______________	______________	_____________
____	________________	_______________	______________	_____________
____	________________	_______________	______________	_____________
____	________________	_______________	______________	_____________
____	________________	_______________	______________	_____________

Analysis: Part One

1. The sine of the elevation angle will be

$$\sin\theta = \textbf{Elevation / width between legs of the air track} = \textbf{Elevation (}\text{in meters}\textbf{) / 1. 00 m}.$$

2. The initial velocity, v_o , will be the **flag width / Time #1**
3. The final velocity, v, will be the **flag width / Time #3**
4. The elapsed time, t, will be **Time #1 + Time #2**
5. Complete the table below and calculate the acceleration using $v = v_o + at$

*(We assume, from the beginning, that positive displacement, velocity and acceleration are **down the hill**.)*

$\sin\theta$	v_o	v	t	a
________	_________	__________	__________	_________
________	_________	__________	__________	_________
________	_________	__________	__________	_________
________	_________	__________	__________	_________
________	_________	__________	__________	_________

6. Construct a graph, named Graph #1, and find the slope of the resulting straight line.
   a) Make the horizontal axis ***sine of inclination angle***
   b) Make the vertical axis ***acceleration*** in m/sec^2
   c) Plot the points.
   d) The result is a straight line.
   e) Autoscale from zero on both axes to show the origin at (0, 0).
   f) Set the maximum value of ***sine of inclination angle*** to 1.00.
   g) Fit the data to a straight line through the origin (proportional).

   Theory shows that the line has a y-intercept of zero. Therefore, it must go through the origin of the graph. (After the fit is obtained, *set the maximum vertical axis value to a number slightly greater than the slope of the line*.)

7. The net force on the cart down the track is **mg sin θ**, therefore, in the absence of friction, $\mathbf{F_{net} = mg \sin \theta = ma}$. When the track is vertical **sin θ** equals 1.00. Therefore, ***a*** = acceleration of gravity = **g** when sin θ = 1.00. By examining either the slope of the line or the unit intercept you can obtain an estimate of the value of **g**.

   Examine your graph. From the graph, determine the ***acceleration*** when the ***sine of the inclination angle*** is 1.00:

   Straight-line fit to $\mathbf{a} = \boldsymbol{g * \sin \theta}$ at θ = 90 degrees: slope = **a** (90º) = **g** = _____________ m/s^2

   And the unit intercept = **a** (90º) = **g** = _____________ m/s^2

   The true value of **g** (in Fort Worth, TX) = 9.795 m/s^2

   %Error (in estimated value of **g**) = [|estimated **g** – true **g** | / true **g**] x 100% = _______________ %

Part Two: Coefficient of Static Friction

1. Place a variety of objects on the track and then elevate the track until they just begin to slide (not roll).
2. In Data Table II, record the height of the track at its steepest position where the object does not slide.

Data Table II

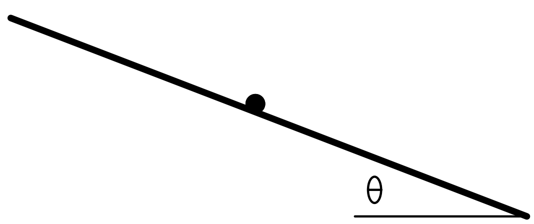
Material or Object	Elevation (centimeters)	sin (Angle of inclination)	Angle of inclination (º)
			º
			º
			º
			º
			º

Analysis: Part Two

1. There are four forces (or force components) acting on an object as it sits on the plane.

I.	**mg sin θ** down along the plane	II	**mg cos θ** into the plane
III	Friction up along the plane	IV	Normal force out of the plane

Draw a simple force diagram (free-body diagram) showing these four forces acting on an object.



2. We know the parallel forces cancel out because the object is not moving. Therefore, at the maximum angle reached before the object begins to move *(study this derivation carefully; 'cause it's amazin!)*

$\mathbf{mg\ sin\ \theta}$ = friction force = $\mu_S \mathbf{F_N}$ and $\mathbf{F_N = mg\ cos\ \theta}$

so $\mathbf{mg\ sin\ \theta} = \mu_S\ \mathbf{mg\ cos\ \theta}$,

mg cancels, and since by definition $\mathbf{sin\ \theta / cos\ \theta = tan\ \theta}$

then use that trigonometric identity to conclude that $\mathbf{\mu_S = tan\ \theta}$

Complete the table below using this result.

Material or Object	**Elevation Angle**	**tan** (Elevation Angle)	μ_S (coefficient of static friction)
	º		
	º		
	º		
	º		
	º		

Part Three: Statics

1. Measure the length of the board (our incline for this part of the experiment). Record it in Data Table III.
2. Find the mass of the cart and record it in Data Table III. Place the cart on the incline.
3. Run a light string from the cart over the pulley and connect the string to a hanging weight. Record the mass of the hanging weight in Data Table III. (*Start with the 50 g hanger as the only hanging weight*.)
4. Adjust the angle of elevation until the cart is balanced (does not move). Record the height of the upper end of the track. The sine of the angle of inclination will be equal to the **measured height / length of incline**.
5. Add a 50 g slotted weight to the hanger and find the new angle that exactly balances the cart.
6. Repeat until you have tested a total of six different hanging masses and recorded all the data in Data Table III.

Data Table III

Length of incline ______ cm (Tape) Mass of cart = **m** = __________ kg (Scale) Weight of the cart = **mg** = __________ N (Use g = 9.81 N/kg)

Mass of hanger and slotted weights(**M**) (in 50 g increments)	Weight of hanger & slotted weights (**Mg**) (N)	Incline height (cm)	sin (Angle of inclination)
50 g	______________	______________	______________
100 g	______________	______________	______________
150 g	______________	______________	______________
200 g	______________	______________	______________
250 g	______________	______________	______________
300 g	______________	______________	______________

Analysis: Part Three

1. When the cart is not moving the force up along the incline equals the force down along the incline, so
   hanger weight = Mg = W = mg sin θ
2. If you graph **hanger weight (Mg)** vs. **sin θ** you should get a straight line with a slope = **mg**, the weight of the cart.
3. Construct this graph, named Graph #2, and find the slope of the best-fit line through the data.
   a) Make the horizontal axis ***sin (inclination angle)*** [no units on this axis]
   b) Make the vertical axis ***hanger weight*** [unit: newtons]
   c) Plot the points. Determine the best line through the origin that fits the data (*proportional*).
   d) Autoscale from zero on both axes to show the origin at (0, 0).
   e) Set the maximum **sin θ**-value to 1.00. Set the maximum **hanger weight**-value to 5.00 N.

The line the computer draws can be ***extrapolated*** to **sin θ** = 1.00 (call this the unit intercept). **sin θ** = 1.00 corresponds to an angle of 90°; i.e. the incline is vertical. The weights of the hanger and the cart must be equal when the board is vertical and they are not moving. The extrapolated cart weight also equals the slope of the line of this graph. Thus, you can get the same information two ways from this graph. (Y***ou can get the weight of the cart from the slope of the line, and from the unit intercept of the line. See for yourself below.***)

The extrapolated **hanger weight** (at sin θ = 1.00) = ______________ N, and two masses, **M** = **m** = ______________kg

The slope provides another estimate of **cart weight** = _______________ N, thus cart mass, **m** = ________________kg.

Therefore, the best estimate of the **cart weight** = _______________ N, thus cart mass, **m** = ________________kg.

%Error in **m** = [|**estimated cart mass – true cart mass| / (true cart mass)] x 100%**. = ________________ %.