Name: ____________________ **Period:** _____ **Due Date:** ___________
Lab Partners: ______________________________ **Launcher Icon:** ___________

Projectiles II - Range & Altitude of Projectile WebAssign

Purpose: To investigate the mathematical description of two-dimensional projectile motion.

Theory: All projectiles follow a parabolic path because the force of gravity accelerates the motion in the vertical dimension downward while the horizontal motion exhibits a constant velocity. We separate these two motions into independent horizontal and vertical equations. (*It's like the graph of* $\boldsymbol{x^2}$ *vs* $\boldsymbol{x}$. *We are plotting* $\frac{1}{2}\boldsymbol{at^2}$ *vs* $\boldsymbol{t}$*, but it is exactly the same concept; you get a parabola in both cases.*)

The general idea is to treat the vertical motion and horizontal motion separately; like two components of a vector. Work only with one component at a time. The common thread that synchronizes them is the time. Frequently, solving both equations for time and then setting the time-independent parts equal to each other allows us to eliminate time from our calculations. When time is not in the equation, we don't have to measure the duration of the flight.

Horizontal Motion Equations:

$a = 0$

Range $= x - x_0 = v_{xo}\, t$

$v_{xo} = v_{muzzle} \cos\theta$

Vertical Motion Equations:

$g = -9.795 \text{ m/sec}^2$ (*Local value of g*)

Altitude $= y - y_0 = v_{yo}\, t + \frac{1}{2} g t^2$

$v_{yo} = v_{muzzle} \sin\theta$

The difficult part about removing time from these equations is that pesky quadratic term in the Altitude equation. The general solution requires use of the quadratic formula, which we would like to avoid at almost any cost. We begin by looking for simplifying conditions.

For example, **if the launch is horizontal**, then $\theta = 0$ and $\sin\theta = 0$, so $v_{yo} = 0$, also. In that case the Altitude equation is much easier to use (*because there is no initial vertical component of velocity*) and finding the time involves merely taking the square root; not solving a quadratic equation. Thus,

Altitude $= y - y_0 = \frac{1}{2} g t^2$

and

$t = [2 (y - y_0) / g]^{1/2}$

Range $= x - x_0 = v_{xo}\, t$

and

$t = [(x - x_0) / v_{xo}]$

Upon algebraically solving both the Range and Altitude equations for **t**, we can set the right sides of these two expressions equal to each other to get

$$(x - x_0) / v_{xo} = [2 (y - y_0) / g]^{1/2} \qquad \text{(Equation 1)}$$

This case is discussed more thoroughly in the Theory Section for Part I. In the Theory Section for Part II you will learn how to handle these equation when the launch is not horizontal.

Theory Section for Part I. Here we consider only the case where a projectile is launched horizontally, thus

$$v_{yo} = v_{muzzle} \sin \theta = 0, \text{ because } \theta = 0.$$

Therefore,

$$\text{vertical displacement} = \mathbf{y - y_o = ½\, g\, t^2 = ½\,(-9.795)\, t^2} = -4.8975\, \mathbf{t^2}.$$

The vertical displacement will always be negative in this case. That simply means the projectile lands below the launch height. The time needed to drop from rest (*There is no initial vertical component of velocity when the projectile is launched horizontally*.) through the specified vertical displacement is

$$\mathbf{t} = [(\mathbf{y - y_o})/(-4.8975)]^{1/2} = [(-\text{launch altitude})/(-4.8975)]^{1/2} = [(\text{launch altitude})/(4.8975)]^{1/2}$$

Both the numerator and denominator in the ratio are negative numbers, so we can just a well treat the negative vertical displacement as a positive launch altitude and divide it by +4.8975, as shown.

Plugging this expression for time into the Range equation (*range in this case means distance to the point where the altitude equals zero*) yields (*recognizing that* $v_{muzzle} = v_{xo}$, *because the projectile is launched horizontally*).

$$\mathbf{Range = x - x_o} = v_{xo}\, [(\mathbf{y - y_o})/(-4.8975)]^{1/2} = v_{muzzle}\, [(\text{launch altitude})/(4.8975)]^{1/2}$$

Notice that we exerted some effort to eliminate the time from this equation. It simplifies our measurements in that we will not need to measure the time it takes the projectile to reach the point where its altitude is zero. If all we really want to know is the muzzle velocity, v_{muzzle}, the all we need to measure are two distances, the launch altitude and the range. In other words, the only tool we need in order to make our measurements is a meter stick or tape measure.

After solving the relevant range equation for the muzzle velocity, we can use the measured launch altitude and the measured range to find the muzzle velocity of the launcher.

$$v_{muzzle} = (\mathbf{x - x_o})[(-4.8975)/(\,\mathbf{y - y_o})]^{1/2} = (\mathbf{Range})\, [(4.8975)/(\text{launch altitude})]^{1/2}$$

This equation offers an alternative method to the two-step method outlined on the next page in Part I of the experimental procedures. This method avoids the intermediate time calculation. You should use this method to confirm the results of Part I after following the two-step method adopted in the experimental section of Part I.

Experimental Procedure

Part I: Finding the Muzzle Velocity of the Launcher using a Horizontal Launch.

1. Fire the ball horizontally (*at* **0º** *relative to the horizon*) and land it on the tabletop. Use the medium (*middle*) setting on the launcher. Find a consistent value for the range, $(x-x_o)$. Be extra careful with this step.

2. Record the initial height of the ball above the tabletop's landing elevation, $-(y-y_o)$.

3. Record the horizontal distance of the flight of the ball, $(x-x_o)$.

4. Repeat two or three times until a consistent distance $(x-x_o)$ is obtained.

Enter the range and altitude of you projectile along with the numerical results of the following calculations into the appropriate blanks at the bottom of this page. Answer each discussion question in the blank section following the question.

5. Calculate the drop time using $(y-y_o) = ½ g t^2$.
   Q1: Where does this equation come from?

Q1: ____

____

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6. Calculate the horizontal component of the initial velocity, v_{xo}, using the range equation, $x-x_o = v_{xo}\,t$. (*In our experimental arrangement* $v_{muzzle} = v_{xo}$ and $v_{yo} = 0$.
   Q2: a) Why is it true to say that $v_{muzzle} = v_{xo}$

Q2:a) ____

____

Q2: b) Why is it true that $v_{yo} = 0$?)

Q2:b) ____

____

Data: Drop Height = $(y-y_o)$ **= _________ m Measured Range:** $x-x_o$ **= ___________ m**

Calculations: drop time = t = _________ s $v_{xo} = v_{muzzle}$ **= _______________ m/s**

7. Enter your Part I-value of v_{muzzle} into the Data and Results Table on the last page.

Theory Section for Part II. If we consider only projectiles that are launched and land at the same altitude, then $\mathbf{y - y_0 = 0}$. Solving the altitude equation for time in this particular circumstance yields:

$$t = -\frac{2v_{y0}}{g} = -\frac{2v_{muzzle}\sin\theta}{g} = \frac{2v_{muzzle}\sin\theta}{|g|} \qquad \textit{Equation II}$$

Substituting this expression for time into the range equation yields the Range($\boldsymbol{v_{muzzle}}$, θ):

$$Range = v_{muzzle}\cos\theta\left[\frac{2v_{muzzle}\sin\theta}{|g|}\right] = \left[\frac{v_{muzzle}^2\sin(2\theta)}{|g|}\right] \quad [\textit{Note}:\ 2\sin\theta\cos\theta = \sin(2\theta)] \qquad \textit{Equation III}$$

This form of the range equation predicts that the maximum range occurs at a launch angle of 45º. At angles smaller than 45º the projectile falls short because it flies too low. At angles greater than 45º the projectile falls short because it flies too high. This conclusion assumes that air resistance is not a factor affecting the flight of the projectile. Our equations model projectiles that fly at low speeds where air resistance has a negligible influence. The motion of baseballs, golf balls and other objects that travel at high speed are not well described by the range equations derived here.

Experimental Procedure

Part II: Finding the Muzzle Velocity of the Launcher using a Variety of Launch Angles.

General Instructions:

1. Place a pile of books on the table so that the top of the books is level with the bottom of the ball in the launch position. (*Note: the position of ball in the launch position is printed on the side of the launcher for your convenience.*)
2. Carefully set the cannon at the correct launch angle and fire the ball. Find the approximate position of the landing zone at the same altitude as the launch altitude. Move the pile of books to the correct location for a landing.
3. Fire the ball a second time and measure the horizontal distance from the launch position to the point where the ball landed on top of the books.
4. Record the horizontal distance the ball traveled in the Data and Results Table on the last page.

5. Repeat the procedure for the other angles indicated in the data table. (*Note: Because the landing zones should be nearly the same, it is highly recommended that you measure the complementary launch angles in the order listed in the table: 80° then 10°; 70° then 20°; 60° then 30°; etc.*)

Part II: continued

Create Graph I

Before making the calculations in the Data and Results Table, you must first construct a graph of **Range *vs* θ.** In ***Graphical Analysis***, begin by entering the launch angles in the first column, in the order they appear in the data table. In the second column enter the range you measured for each angle. In the File menu select Settings for Untitled and Degrees, to simplify the trigonometric calculations used later. Turn off the connecting lines.

According to *Equation III*, the range should be proportional the sine of 2θ. That means it will cross the horizontal axis at 0º and 90º, and will reach its peak at 45º. Make sure all the data points from 0º to 90º, including the peak, are visible on your Graph. Fit the data to the following defined function,

(V^2)*sin(2*x)/9.795

When you use the Define Function button you must not use quotation marks, so in this equation the x refers to θ, because θ is plotted on the horizontal axis; Make sure the column you named Theta has the short name, x. In this equation V is the only unknown parameter. ***Graphical Analysis*** will find the best value of V that fits the data. Ignore the negative sign of V, if you get one; it is due to a quirk in the software. Since V = v_{muzzle}, this graph gives you a second estimate for the muzzle velocity. Enter this graphical value of v_{muzzle} into the Data and Results Table on the last page.

Create Graph II

Before making the calculations in the Data and Results Table, you must also construct a graph of **Range *vs* sin (2θ).** In the Data menu create a new "calculated" column and use it to compute **sin (2θ)**. In the page menu add a new page, which is a copy of the current page. Graph I will be on page I and Graph II will be on page II. In Graph II, graph the range on the vertical axis and **sin (2θ)** on the horizontal axis.

According to *Equation III*, this graph should be a straight line through the origin. Fit this line to the defined function

V^2*x/9.795

This will give you a third estimate of v_{muzzle} since V = v_{muzzle}. Enter this value of v_{muzzle} into the Data and Results Table on the last page.

Which of the three estimates of muzzle velocity does the best job of fitting the range data in the Data and Results Table? (*Is there a best choice in your case? If so, which? If not, why not?*)

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Theory Section for Part III. If you wish to consider the general projectile motion problem, then we would probably want to know both the x and y positions as functions of time. These are given on page 1, thus

$$(x, y) = (v_{xo}\,t,\ y_o + v_{yo}\,t + ½\,g\,t^2) = (v_{muzzle}\,(\cos\theta)\,t,\ y_o + v_{muzzle}\,(\sin\theta)\,t - 4.8975\,t^2)$$

Given a series of times, you could easily use ***Graphical Analysis*** to calculate the x and y coordinates and plot these pairs of coordinates on an xy-graph.

Experimental Procedure

Part III: Calculating the Range and Maximum Height of the Trajectories.

Calculations:

1. Calculate the range of the projectile at each launch angle using the range equation (*Equation III*) with all three of your estimates of v_{muzzle}.
2. Find the percent difference in your experiment between the calculated and measured range at each angle. Use the calculated range as the reference value.

%Difference = 100% x absolute value [(calculated range - measured range) / calculated range]

3. The maximum projectile height at each launch angle can be calculated using

$$v_{yo} = v_{muzzle} \sin\theta \qquad \text{and} \qquad v_y^{\,2} = v_{yo}^{\,2} + 2\,g\,(y - y_o)$$

Remember that $v_y = 0$ at the top of the arc and that $g = -9.795\ m/s^2$ at all points along the trajectory. This means that $(y - y_o) = (y - y_o)_{MAX}$ when $v_y = 0$.

4. Solve these two equations for $(y - y_o)_{MAX}$, and write the equation for the maximum height of the projectile along its parabolic trajectory as a function of the muzzle velocity and the launch angle. (*Derive the equation you will use and write it here*.)

$(y - yo)_{MAX}$ = ______________________________

Use the muzzle velocity from Part I, the muzzle velocity from Graph I, and the muzzle velocity from Graph II to calculate the maximum height in the last three columns of the Data and Results Table.

Optional Extra Credit Section

Given a series of times, you could easily use ***Graphical Analysis*** to calculate the x and y coordinates and plot these pairs of coordinates on an xy-graph according to the functions described in the Theory Section for Part III.

For an optional extra credit exercise after you've completed the rest of this assignment, you may prepare such a graph. Create a column of time values from 0 to 10 seconds at intervals of 0.1 s. Use the times in that column to create a column of x-positions and another column of y-positions corresponding to each time. (*The instructor will assign you a specific launch angle if you are interested.*)

Set $\boldsymbol{y_o}$ = 70 m and $\boldsymbol{v_{muzzle}}$ = 25 m/s, then graph **y vs x**. Turn OFF the connecting lines. Fit the graph to a parabola. Each interested student must prepare this graph without assistance from any other student. Name this Graph III.

Only correctly prepared and original graphs will receive any extra credit.

Data and Results Table

v_{muzzle} = ____________ m/s (from **Part I**); v_{muzzle} = __________ m/s (from **Graph I**); v_{muzzle} = __________ m/s (from **Graph II**)

			(Using v_{muzzle} Part I)		(Using v_{muzzle} for Graph I)		(Using v_{muzzle} from Graph II)		(Using v_{muzzle} from each part) Part I	Graph I	Graph II
Trial	**Launch Angle** θ *(deg)*	**Measured Range** $x-x_0$ *(m)*	**Calculated Range** $x-x_0$ *(m)*	**%Diff**	**Calculated Range** $x-x_0$ *(m)*	**%Diff**	**Calculated Range** $x-x_0$ *(m)*	**%Diff**	**Max Height** $y-y_0$ *(m)*	**Max Height** $y-y_0$ *(m)*	**Max Height** $y-y_0$ *(m)*
1	80º	______	_______	_______%	_______	_______%	_______	_______%	_______	_______	_______
2	10º	______	_______	_______%	_______	_______%	_______	_______%	_______	_______	_______
3	70º	______	_______	_______%	_______	_______%	_______	_______%	_______	_______	_______
4	20º	______	_______	_______%	_______	_______%	_______	_______%	_______	_______	_______
5	60º	______	_______	_______%	_______	_______%	_______	_______%	_______	_______	_______
6	30º	______	_______	_______%	_______	_______%	_______	_______%	_______	_______	_______
7	50º	______	_______	_______%	_______	_______%	_______	_______%	_______	_______	_______
8	40º	______	_______	_______%	_______	_______%	_______	_______%	_______	_______	_______
9	45º	______	_______	_______%	_______	_______%	_______	_______%	_______	_______	_______

For %Difference calculations use the calculated range as the reference value. You will find that the trajectory calculator you created in the computer lab will be helpful in confirming your results in this table. Use it to make sure you have not made any large or obvious errors in your calculations. You could also create a new spreadsheet just for doing these calculations. If you create a spreadsheet for completing this table, you may submit a printout to replace this table provided that all the columns include proper headings, that all columns are in the same order, and that the spreadsheet printout fits on one page.