Name: ________________________________ **Book:** ___ **Period:** ___ **Due Date:** __________

EXPONENTIAL DECAY – H1

Purpose: Introduce the diversity of mathematics involving the Euler Number ***e***. It is the base of the natural logarithms. It is a number that arises quite naturally from the study of numbers and has applications in a wide range of mathematical and scientific settings. Familiarize yourself with the various forms used to express the exponential function. Learn a bit about Euler and the history of this important mathematical constant. Prove Euler's Equation.

Who put the *e* in the Exponential Function?

e is an irrational, real number. Like π it is an infinite, non-repeating decimal, which simply means that it cannot be calculated as the ratio of two integers. ***e*** is a constant that appears in numerous kinds of mathematical models. Examples of such models are those involving growth (*including compound interest*) or decay, the statistical "bell curve," the shape of a hanging cable (*or the Gateway Arch in St. Louis*), certain problems of probability, some counting problems, and even the study of the distribution of prime numbers.

One slightly obscure example: it also appears with π in Stirling's Formula for approximating factorials.

$$\sqrt{2\pi}\, n^{n+1/2} e^{-n+1/(12n+1)} < n! < \sqrt{2\pi}\, n^{n+1/2} e^{-n+1/(12n)}$$

The table at the right shows a comparison of the accessible factorials using Stirling's Formula in Excel and comparing those with the first twenty factorials according the FACT function in Microsoft Excel. Stirling's Formula does a terrible job with 0!, which by definition must equal one, but it gets better as n increases.

Stirling's Formula really shines at very large n, where the logarithmic form is more often and more easily employed.

$$\ln n! \approx n \ln n - n.$$

Very large factorials quickly exceed the capacity of most computers, but the logarithmic form of Stirling's Formula lets computers continue through a calculation containing the factorial function to a conclusion without overflowing any registers.

Very large factorials are often encountered in disciplines such as Statistical Mechanics, where the behavior and interactions among large numbers of atom-sized particles are modeled.

n	n! (*Stirling*)	< n! (*Excel*) <	n! (*Stirling*)
0	0.000000E+00	1.000000E+00	0.000000E+00
1	9.958702E-01	1.000000E+00	1.002274E+00
2	1.997320E+00	2.000000E+00	2.000652E+00
3	5.996096E+00	6.000000E+00	6.000599E+00
4	2.399082E+01	2.400000E+01	2.400102E+01
5	1.199699E+02	1.200000E+02	1.200026E+02
6	7.198722E+02	7.200000E+02	7.200092E+02
7	5.039335E+03	5.040000E+03	5.040041E+03
8	4.031589E+04	4.032000E+04	4.032022E+04
9	3.628506E+05	3.628800E+05	3.628814E+05
10	3.628560E+06	3.628800E+06	3.628810E+06
11	3.991461E+07	3.991680E+07	3.991688E+07
12	4.789794E+08	4.790016E+08	4.790024E+08
13	6.226774E+09	6.227021E+09	6.227029E+09
14	8.717531E+10	8.717829E+10	8.717838E+10
15	1.307635E+12	1.307674E+12	1.307675E+12
20	2.432861E+18	2.432902E+18	2.432903E+18
50	3.041401E+64	3.041409E+64	3.041409E+64
100	9.332615E+157	9.332622E+157	9.332622E+157
120	6.689500E+198	6.689503E+198	6.689503E+198
142	2.695363E+245	2.695364E+245	2.695364E+245

The logarithmic form of Stirling's Formula can handle factorials in the range of Avagadro's number very easily.

Q: Use your calculator to estimate ln n! where n = $6.02 \times 10^{+23}$.

ln [($6.02 \times 10^{+23}$)**!**] = ________________________________

The exponential function shows up frequently in calculus; whenever logarithms are encountered. No surprise there! Its frequent appearance in other areas might be a little surprising, but should not be. When ***e*** serves as the base in an expression like Ae^x, it takes its place among all possible bases ***b*** in the general expression Ab^x. The defining feature that sets ***e*** apart from the other bases has to do with its derivative (*slope of the tangent line of the equation y =A b^x*). The derivative of b^x is always proportional to b^x. One of the many unique features of ***e*** is that in the derivative of <u>Ae^x</u> the proportionality constant is exactly 1.000…. Thus,

$$d/dx(A\boldsymbol{b}^x) = P\times \mathbf{A}\boldsymbol{b}^x, \text{ where P is a proportionality constant; } P = 1 \text{ if and only if } \boldsymbol{b} = \boldsymbol{e}$$

e is classically defined by the following equation:

$$\boldsymbol{e} = \lim_{n\rightarrow infinity} (1 + 1/n)^n.$$

Although this definition approaches the limiting value very slowly, it does the job elegantly. You can check this on your calculator or in a spreadsheet. The true value of ***e*** is approximately 2.718281828459045...

The table at the right shows a sample of estimates of ***e*** using values up to n = 10 million. Even though you have to use large values of the index, n, you do not have to compute all the earlier ones to get an answer. There are better ways to calculate ***e***, but this table demonstrates that the original definition works.

n	e^n	%Error
1	2.0000000	26.4%
2	2.2500000	17.2%
3	2.3703704	12.8%
4	2.4414063	10.2%
5	2.4883200	8.46%
10	2.5937425	4.58%
20	2.6532977	2.39%
30	2.6743188	1.62%
40	2.6850638	1.22%
50	2.6915880	0.982%
100	2.7048138	0.495%
200	2.7115171	0.249%
300	2.7137652	0.166%
400	2.7148917	0.125%
500	2.7155685	0.0998%
1,000	2.7169239	0.0500%
2,000	2.7176026	0.0250%
3,000	2.7178289	0.0167%
4,000	2.7179421	0.0125%
5,000	2.7180101	0.0100%
10,000	2.7181459	0.00500%
100,000	2.7182682	0.000500%
1,000,000	2.7182805	0.0000500%
10,000,000	2.7182817	0.00000495%

Q: Use your calculator to estimate ***e*** using n = 1,000,000,000.

e ≈ ____________________; %Error =______________%

In the 1720's, the Swiss mathematician Leonhard Euler was the first to study the number ***e***. Its existence was first implied in the work of John Napier, the inventor of logarithms, in 1614. Euler, in 1727, was the first to use the letter ***e*** to stand-in for this value even though we cannot write it out explicitly. As a result, ***e*** is sometimes called the Euler Number, the Eulerian Number, or Napier's Constant. It is never, however, called Euler's Constant. That is a different constant symbolized by the letter gamma (γ).

We have Euler to thank for the following notational conventions: *f(x)* for a function (*1734*), ***e*** for the base of natural logs (*1727*), ***i*** for the square root of -1 (*1777*), π for pi, Σ for summation (*1755*), the notation for finite differences, ***Δy***, and many others. Euler was such a prolific mathematician that the **St Petersburg Academy**, where Euler started and ended his academic career, with a stint in between at the **Berlin Academy**, needed 50 years after his death to complete its publication of his collected papers.

Without belaboring the point, you get the basic idea. Both Euler and the Euler Number are ubiquitous throughout mathematics and therefore throughout the sciences. He and it will pop-up in surprising places. Usually without warning. If you ever work in any technical or scientific field, he and it will be hard to avoid.

There is one final equation involving ***e*** that we must discuss before we proceed to the topic of this lab. It is so startling that almost everyone, who sees it for the first time, says or wants to say something like "Where did that come from?"

It seems unreal at first glance and only begins to make some sense once we've seen the proof. The proof is so elegant and concise it is a wonder that it doesn't seem obvious. Like many clever proofs in mathematics, it begins with a statement that doesn't seem to be very promising. However, by the time the proof is completed, which isn't very long, we are startled that it takes so few steps.

Here is the equation. It provides an intriguing connection between ***e***, the trigonometric functions, and the complex numbers. It is known as the Euler Equation.

$$e^{xi} = \cos(x) + \sin(x)\, i$$

How does the base of the natural logarithms tie in with the trigonometric functions and the square-root of minus one? The proof follows.

<u>Proof of Euler's Equation</u>. For those who have never seen if before, this is a relatively simple proof. If you have had or even seen a little calculus, this proof should easily make sense. Even if you haven't had calculus, it should be more that 90% comprehensible because of the simplicity of the algebra. Just read carefully.

Begin with the parametric equation defining a particular family of complex numbers z in terms of parameter x.

$$z = \cos(x) + \sin(x)\, \boldsymbol{i}$$

The domain of x is all real numbers, but the domain [0 to 2π] is sufficient to complete the circle once around.

On the complex plane this equation defines a locus of points arrayed around the circumference of a unit circle centered at the origin of the complex plane. Cos(x) is the real-coordinate of points on the circle and sin(x) is the complex-coordinate of points on the circle. The value of x must be unit-less and the trigonometric functions are then evaluated with x in the unit-less-unit radians. [*For x to be unit-less in a scientific context, it might be a ratio of two quantities with the same units (like the ratio of t/τ which has units of s/s = unit-less), or it could be the product of two quantities with inverse units (like the product λ*t which has units of s^{-1}*s = unit-less).*]

While we're here, notice that when x = 0, z = 1 + 0 $\boldsymbol{i}$ = 1.

Now differentiate z with respect to x; (*when taking the integral or derivative* $\boldsymbol{i}$ *acts just like any other constant.*)

$$dz/dx = -\sin(x) + \cos(x)\, \boldsymbol{i}$$

Substitute $\boldsymbol{i}^2$ for –1.

$$dz/dx = \sin(x)\, \boldsymbol{i}^2 + \cos(x)\, \boldsymbol{i} = \cos(x)\, \boldsymbol{i} + \sin(x)\, \boldsymbol{i}^2$$

Factor out an $\boldsymbol{i}$.

$$dz/dx = [\cos(x) + \sin(x)\, \boldsymbol{i}]\, \boldsymbol{i}$$

Substitute z for cos(x) + sin(x) $\boldsymbol{i}$.

$$dz/dx = z\, \boldsymbol{i}$$

Separate the variables z and x.

$$(1/z)\, dz = \boldsymbol{i}\, dx$$

Integrate both sides of the equation

$$\int (1/z)\, dz = \int \boldsymbol{i}\, dx$$

Yielding this expression, for some constant C, by indefinite integration.

$$\ln(z) = x\, \boldsymbol{i} + C$$

Now, use the fact that when x = 0, z = 1. We see that ln(z) = ln(1) = 0 $\boldsymbol{i}$ +C, since x = 0 when z = 1. But we know from the definition of the logarithm that ln(1) = 0. Therefore, it must also be true that C = 0. C is a constant, so C is zero for all z; not just at this special value of z. Thus, we draw the general conclusion that

$$\ln(z) = x\, \boldsymbol{i}$$

Expanding the logarithm on both sides using the exponential function yields

$$z = e^{x\boldsymbol{i}} = \cos(x) + \sin(x)\, \boldsymbol{i}$$

A consequence of Euler's Equation is that

$$\boldsymbol{e}^{\pi \boldsymbol{i}} = -1$$
$$\boldsymbol{e}^{\pi \boldsymbol{i}} + 1 = 0$$

The Euler Equation is remarkable in that it involves the five most important constants in all of mathematics: 0, 1, $\boldsymbol{i}$, π, and $\boldsymbol{e}$. Plus, it defines relationships among the trigonometric functions, the exponential function, the logarithmic function, and the complex numbers. All this from one equation!

Name: ______________________________ **Period:** ________

EXPONENTIAL DECAY – H2

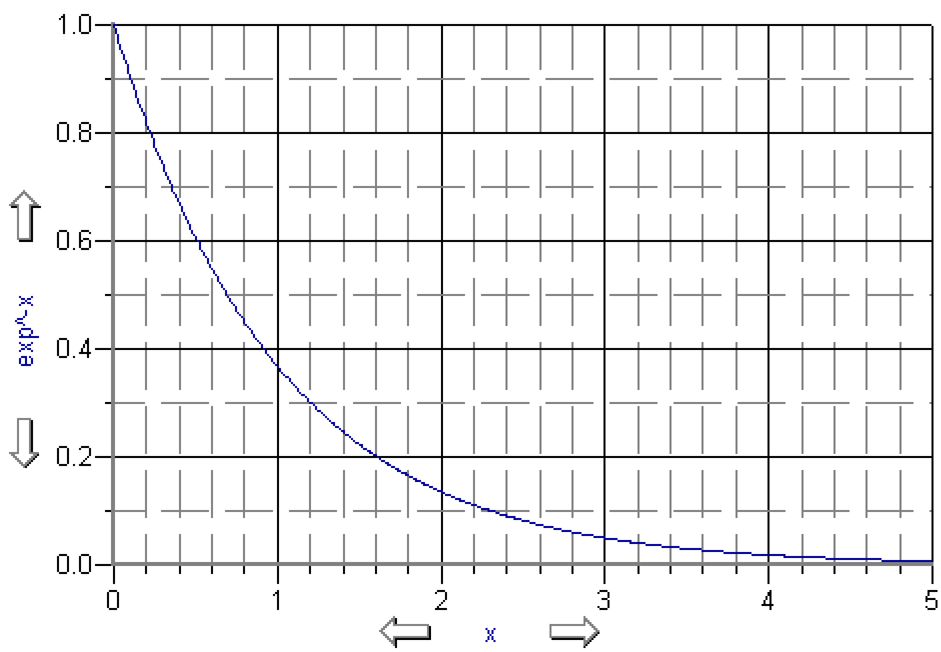
Purpose: Introduce the exponential decay function. Familiarize yourself with the various forms used to designate the exponential decay function.

The Exponential Decay Function:

The exponential decay function is an exponential function with ***e*** as the base and a negative exponent. ***e*** is the base of the natural logarithms. There are three ways to write the exponential function. The first and third work best when the function must be written as a line of text without subscripts or superscripts; like a formula in a spreadsheet or ***LoggerPro 3***, for example.

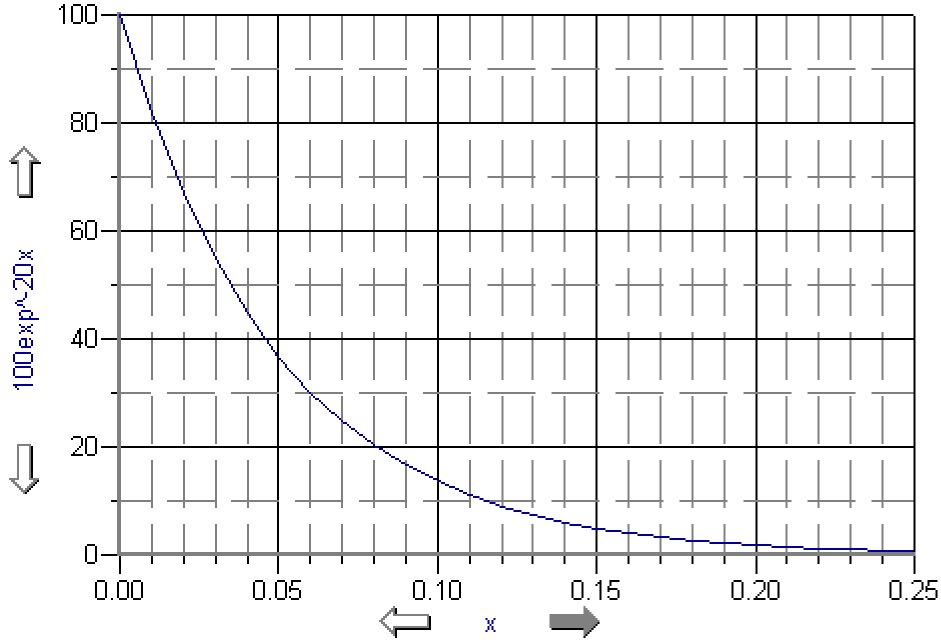
$$Y = A\ e\text{^}(-Bx) = A\ e^{-Bx} = A\ \exp[-Bx]$$

Natural systems that can be modeled using the exponential decay function may differ in their details, but mathematically they all look the same. The parameter A indicates where the function crosses the y-axis. The parameter B determines how quickly the curve drops toward the x-axis. The curve approaches the x-axis asymptotically. (*It never drops below zero.*) You can think of A and B as stretching or shrinking the exponential function along each axis, but it is often more useful to think of the exponential curve as constant while the x and y axes shrink or stretch to accommodate the parameters A and B.



If A and B are both 1, then the graph of the exponential function looks like this graph. When B = 1, the graph falls to 1/***e*** of its initial value, namely 0.367879441, at x = 1 on this graph.

If A is increased to 100, then the vertical axis must be relabeled from 0 to 100. If B is increased to 20, then the horizontal axis must be re-labeled from 0 to 5/20 = 0.25. After re-labeling both axes, the curve will again look that same as it does in this figure. See the next page.



If A equals 100 and B equals 20, then you get this graph of the exponential decay function. In this graph, Bx = 1 when x = 1/20 = 0.05. The value of the function is then equal to 100/***e*** or 36.78794412

Notice that the curves in these two graphs have the same shape. Only the axis scales have been changed. The exponential decay function has this wonderful property; that its basic shape remains invariant under the influence of these stretching operations.

The parameter A is the vertical scaling factor. Its value indicates the starting value of the measured quantity (*the starting value is often symbolized as* N_0). It is the initial number or the initial reading at whatever time you decide to start making measurements. If we are talking about a population of antelope, for example, $\mathbf{N_0}$ is the number of antelope at the time you start studying the antelope population. The fact that the population of antelope has been in decline for decades before you started studying it does not affect the exponential function. If you had started decades earlier or decades later, the exponential decline function would yield the same curve, assuming that the population is in an exponential decline.

The parameter B can be interpreted as a measure of how fast the population is dropping. In most population studies, whether antelope or atomic nuclei, x is taken as time. 1/B is then a measure of the time it takes the population to drop to 1/***e*** of its initial value. 1/B is known as the time constant (*the usual symbol for the time constant is* τ). B is known as the rate constant (*the usual symbol for the rate constant in nuclear systems is* λ*, but in other cases the letter* ***k*** *is more commonly used*). λ is inversely related to the probability (P) that a given member of the population will survive for a given amount of time (*small* λ *means a high probability of survival, while a large* λ *means small probability of survival*). If you are plotting the data in seconds, λ is inversely related to the probability of surviving for one more second. If you are plotting in time units of months, then λ is inversely related to the probability of surviving for one more month.

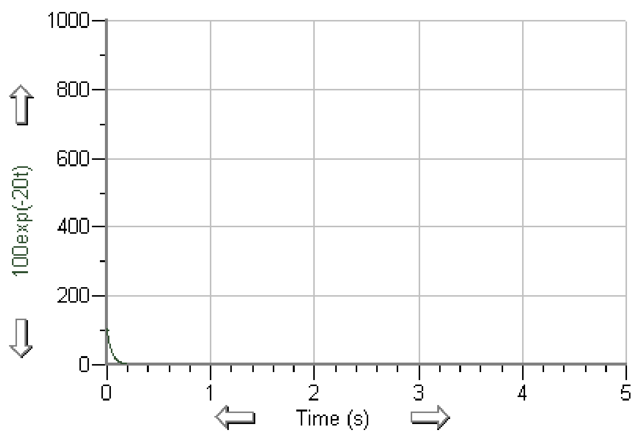
The half-life of the population is the time it takes for the population to fall to one-half of its initial value. However, it turns out that you can start anywhere on the graph and find the same half-life. On the graphs above, it takes about 3-1/2 small units to drop to 50% of the initial value.

For fun, find out how long it takes to drop from 40% of the initial value to 20% of the initial value. Then, find the time it takes to drop from 60% of its initial value to 30% of its initial value. If you try it, you will see that all the half- lives are really the same half-life. Half-life, like τ, is a well-defined time for the process being modeled by an exponential function. It always has the same value for a given decay process.

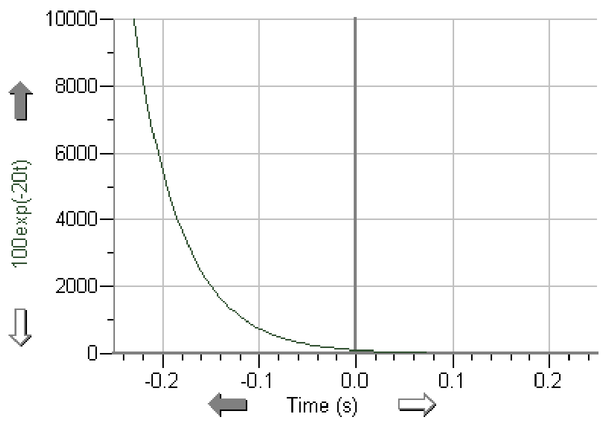
If the probability for survival for the next unit time interval does not change with time, then B does not change with time, and then the system is a good candidate to be modeled by the exponential decay function.

Changing Views of the Exponential Decay Function

Some graphs of the exponential decay function may seem to violate the observation made earlier that all exponential decay functions have the same shape. This difference is more apparent than real. The differences have to do with choices made for the range of values covered by each axis. The following four graphs are views of the same exponential decay function, $y = Ae^{-Bt}$. In each case A = 100 and B = 20. Note how the changes in the scales make them appear to be different. Note also the similarities. You need to be able to recognize this function no matter how the scales are adjusted.



The changes in scale shown in the figures on this page give you some idea of how different the exponential function can look because of scale distortions.



The derivative of the exponential decay function, given that

$$N = N_0\, e^{-\lambda t}$$

Is given by

$$dN/dt = d/dt[N_0\, e^{-\lambda t}] = -\lambda\, [N_0\, e^{-\lambda t}] = -\lambda\, N$$

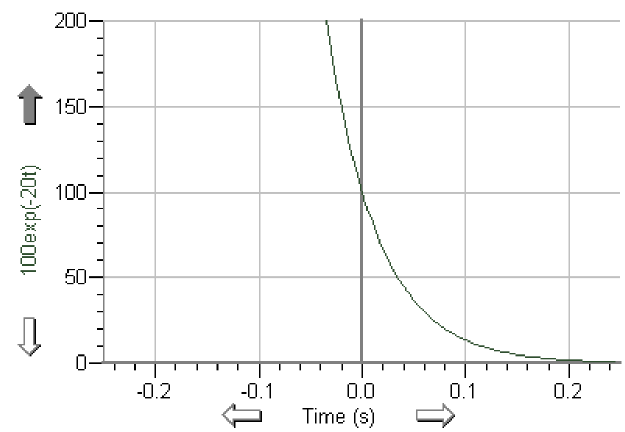
This says that the number remaining, N, is declining (*negative sign*) at a rate that is proportional to N. The rate constant, λ, is the proportionality constant.

If you change the scale on any exponential decay curve to the following settings:

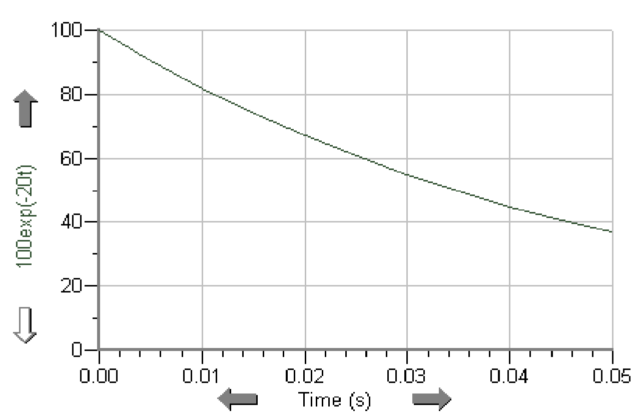
Scale the x-axis from 0 to 5/B

Scale the y-axis from 0 to A

Then you will get curves that look exactly like those on the two previous pages.



Your laboratory graphs will have four curves on them from four different systems. Use the scale that is appropriate for the curve with the smallest value of B. In the lab itself the parameter λ plays the same role as B does here.



Name: ______________________________ **Period:** ________

EXPONENTIAL DECAY – H3

Purpose: Introduce the basics of nuclear decay and introduce the mathematical and graphical techniques we will be using in this laboratory exercise.

Nuclear Decay - Theory:

(*This section ignores nuclei inside an atomic bomb, or a nuclear reactor, or a particle accelerator, or a star, like the Sun, where the environmental conditions are so extreme that they do affect the rate of nuclear decay. Here we only consider spontaneous decay process, not induced decay and not fusion.*)

When an unstable atomic nucleus decays spontaneously, various particles, called alpha or beta particles, are emitted and the nucleus of the atom transforms into a new element. (*A nuclear isotope and its product nucleus are often referred to as parent and daughter nuclei.*) Particles known as neutrinos (ν_e, *and anti-neutrinos,* $\underline{\nu}_e$) are also sometimes emitted. (*Neutrinos balance the quantum numbers for spin and parity in these decays.*) Here are some examples. (*Other less common decay modes are also known but are not common among the natural radioactive elements and, so, are not discussed in detail here.*)

$^{m+n}_{n}X \rightarrow {}^{m+n-4}_{n-2}Y + {}^{4}_{2}He + \text{energy}$ (*alpha emission – emission of an alpha particle = helium-4 nucleus*)

$$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$$

$^{m+n}_{n}X \rightarrow {}^{m+n}_{n+1}Y + {}^{0}_{-1}\boldsymbol{e} + \underline{\nu}_e + \text{energy}$ (*beta-minus emission – emission of a beta particle = electron*)

$$^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}\boldsymbol{e} + \underline{\nu}_e$$

$$^{137}_{55}Cs \rightarrow {}^{137}_{56}Ba + {}^{0}_{-1}\boldsymbol{e} + \underline{\nu}_e$$

$^{m+n}_{n}X \rightarrow {}^{m+n}_{n-1}Y + {}^{0}_{1}\boldsymbol{e} + \nu_e + \text{energy}$ (*beta-plus emission – emission of a positron = anti-electron*)

$$^{1}_{1}p \rightarrow {}^{1}_{0}n + {}^{0}_{1}\boldsymbol{e} + \nu_e$$

$$^{22}_{11}Na \rightarrow {}^{22}_{10}Ne + {}^{0}_{1}\boldsymbol{e} + \nu_e$$

$^{m+n}_{n}X + {}^{0}_{-1}\boldsymbol{e} \rightarrow {}^{m+n}_{n-1}Y + \nu_e + \text{energy}$ (*electron capture – nuclear capture of an inner shell electron*)

$$^{1}_{1}p + {}^{0}_{-1}\boldsymbol{e} \rightarrow {}^{1}_{0}n + \nu_e$$

$$^{7}_{4}Be + {}^{0}_{-1}\boldsymbol{e} \rightarrow {}^{7}_{3}Li + \nu_e$$

The daughter nucleus is frequently created in an excited rotational state and loses rotational energy, after a brief delay, by emitting one or more photons of electromagnetic energy, usually in the gamma ray (*these are the very highest energy photons*) region of the electromagnetic spectrum.

The nucleus can also lose rotational energy by literally bumping one of the inner shell electrons surrounding the nucleus. This is usually sufficient to completely eject the electron from the atom. Subsequently, one of the remaining electrons surrounding the atom will fall into the vacant orbital and emit an x-ray photon in the process. This process is known as internal conversion and must not be confused with beta-minus emission. (*Beta minus emission changes the atomic number of the nucleus; internal conversion transfers energy from the nucleus to the surrounding electrons without changing the atomic number of the nucleus.*)

Excited nuclei are also known to emit a proton. This is known as beta-delayed proton emission. Nuclei especially rich in either protons or neutrons can emit either a proton or a neutron; these decay modes are known as proton emission and neutron emission, respectively.

Dynamic, and incompletely understood, forces within the nucleus govern the internal processes that lead to nuclear decay. Forces from outside the nucleus do not disturb these internal forces. The key feature of interest is that the <u>probability</u> that a given nucleus decays within a specified time interval <u>never changes</u>.

The odds of decay occurring in the next second do not increase just because the nucleus has been around a long time. The character of the forces at work within a nucleus are unaffected by such things as the temperature and pressure of normal Earthly conditions. The presence or absence of other atoms, likewise, has no effect. Even the presence of daughter nuclei from prior decays does not affect the internal dynamics of the remaining nuclei.

Since no forces within the nucleus are influenced by external events, the probability of these forces inducing a nuclear decay will not change either. The proof that this picture of the nucleus is correct comes from monitoring the number of atoms that remain as a function of time and comparing the changing numbers with the exponential function's theoretical prediction.

There is a <u>fixed, invariant, and unwavering probability</u> (*independent of the environment, independent on how many nuclei are present, and independent of how many nuclei have already decayed*) that a nucleus will decay in a specific amount of time. This probability for a given isotope does not change with time.

Example - Imagine the probability of a certain nucleus decaying in the next second is 1/10,000,000. If you have 10^{+20} atoms in your collection (*about 1/6th of a millimole*) then about 10^{+13} of them, on average, should decay in the next second. In the following second, 1 in 10,000,00 will decay. There will be fewer decays in the 2nd second, but only because there are fewer atoms available to decay. The number of decays per second, known as the ***activity***, will thus decline in parallel with the number of atoms. Eventually, the activity will drop to 50% of its initial value. At that point only 50% of the original atoms remain. The activity reaches one-half its initial value at the same moment that the number of atoms reaches one-half of its initial value.

The specific <u>time interval</u> during which the activity and the number of atoms drop by 50% is called the half-life, $\boldsymbol{t_{1/2}}$, of that nuclear isotope. In a large collection of such nuclei, on average, one-half of the nuclei will decay in the first half-life. Half of the remaining nuclei will decay if we wait an equal amount of time again, i.e. a second half-life. If we wait the same amount of time again, i.e. a third half-life, only one-eighth of the original nuclei will remain - and so on. It does not matter when you start; the half-life you measure is always the same for a given isotope.

Probability - If the half-life of some radioactive element is known to be one minute, and we start with one hundred nuclei, then statistically speaking, at the end of the first minute only 50 should remain. At the end of the second minute, only 25 should remain. At the end of the third minute only twelve and one-half should remain. Remember always that these predictions are statistical (*they are called* ***expectation values***). Obviously, we cannot have 12½ atoms. The true values will fluctuate because of the poor statistics with such a small sample. With a much larger sample, 10^{+20} nuclei say, the measurements come much closer to the expectation values of the sample.

Graphing the number of remaining (*un-decayed*) nuclei versus time, or the activity vs time, yields the decay curve for a radioactive isotope. The half-life is characteristic of each specific nuclear isotope and can be determined, from analysis of the decay curve.

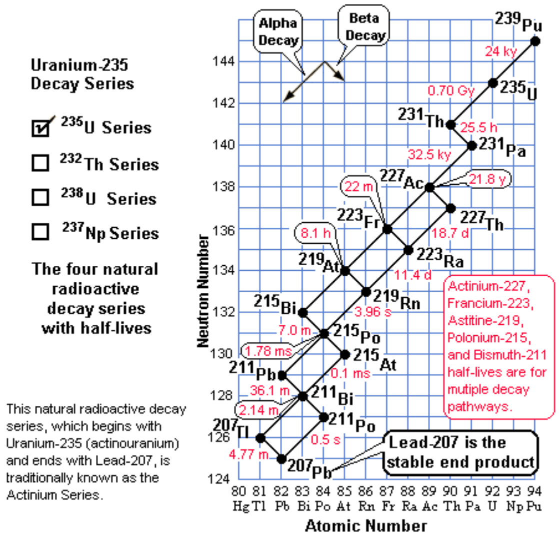
First, a cautionary note: Nuclear decay is seldom as simple and straightforward as this discussion suggests. We are not going to consider any of the complications in lab. However, it might be interesting for you to know that most radioactive decays create radioactive daughter nuclei. In general, therefore, radioactive decay for a whole series of different radioactive nuclei (*the daughters, grand-daughters, great-grand-daughters, etc. of the original parent nuclei*) is found in the same sample at the same time. In real nuclear studies this complicates the analysis beyond what we will be attempting in this lab.

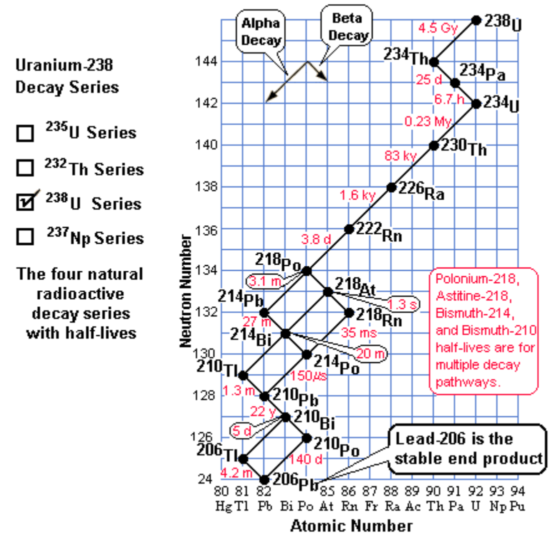
A further cautionary note: Another complication is that some radioactive nuclei can decay by more than one route and thus can produce more than one type of daughter nucleus. While an individual nucleus can only decay by one path, in a collection of identical nuclei some will follow one path and some the other. The daughter nuclei of both pathways will be observed and they will further complicate the analysis.

These and other complications are relatively easy to deal with in the setting of a research laboratory, where a lot of monitoring equipment and computing power is available. We will not be considering them in any detail here, where you are the monitoring equipment and your TI calculator and ***LoggerPro 3***, between them, provide most of the computing power.

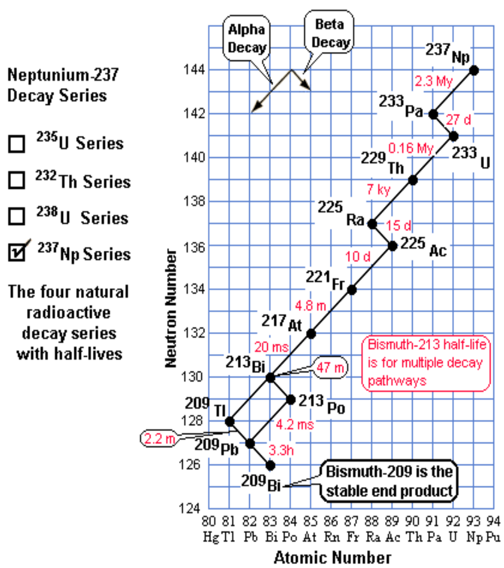
Four Examples of Nuclear Decay Series

These are two well-studied nuclear decay series of the two most common, naturally occurring isotopes of Uranium. Both series end with a stable isotope of lead. The half-lives of the various intermediates range from giga-years (10^{+9} *years*) to fractions of a millisecond (10^{-3} *s*). These half-life differences reflect the range of stabilities of the various isotopes. Note that some isotopes can decay by two possible pathways. Also note that plutonium-239 is not a naturally occurring isotope. Plutonium is manufactured by bombarding Uranium with neutrons. It is a by-product of the nuclear fuel industry and is used to make nuclear weapons.



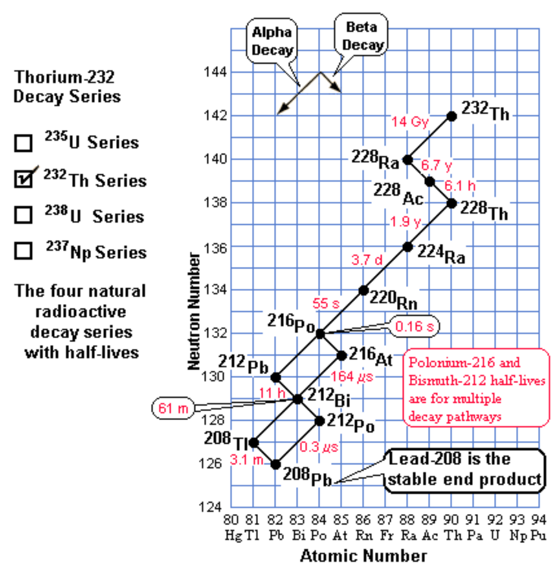


These next two are nuclear decay series for two other naturally occurring radioactive isotopes.



Neptunium-237 is manufactured as a by-product of plutonium production, but it also occurs in very small quantities in natural Uranium ores due to interactions between Uranium atoms and the neutrons emitted by other Uranium atoms within the same ore.

Thorium, on the other hand, is everywhere. It makes up about 0.0007% of the Earth's crust. Its very long half-life indicates that it is not very radioactive and does not pose much of a health risk in the environment. When bombarded by neutrons with the right energy, however, Thorium-232 is converted to Uranium-233, which can in turn be used as a nuclear fuel.



The Exponential Function - The equation that best describes the shape of the decay curve for simple radioactive decay starts with the initial number of nuclei, N_o and calculates the number remaining at later times, N(t), as:

$$N(t) = N_o e^{-\lambda t} = N_o e^{-\mathbf{k}\, t} = N_o e^{-t/\tau} \qquad \textbf{\textit{Equation 1}}$$

The parameter λ *(sometimes a lower case* **k** *is used instead of* λ) is characteristic of the specific nuclear decay or other probabilistic process under study. It is called the rate constant for the process and has units of 1/time = time^{-1} (*1/s = s^{-1} and 1/year = $year^{-1}$, being two common examples*).

This Figure shows the graph of the equation

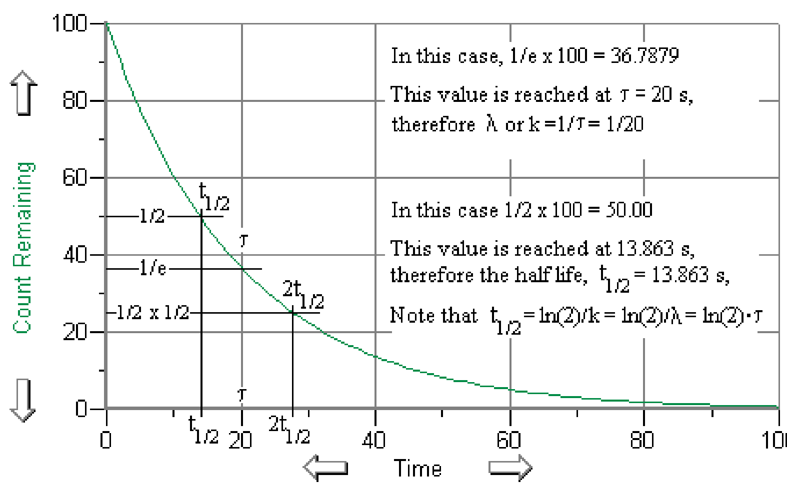
$N(t) = N_0*e$^(–λ*t) = N_0*e^(–k*t) = N_0*e^(–t/τ)

N(t) = 100 exp(-0.05 * t) = 100 exp(-t / 20)

The parameter τ (*known by its various synonyms as the time constant, the relaxation time, the characteristic time, or the lifetime of the process*) is simply the inverse of **λ.**

(τ = 1/k = 1/λ̃).

The time constant, τ, is the amount of time required for the count to fall to 1/***e*** of its initial value. Since the constant ***e*** = 2.718281828…, and $1/e = e^{-1}$ = 0.3678794412… So the lifetime of a process is the time it takes for the initial value to fall to about 36.8% of its initial value.



The function on the right side of ***Equation 1*** is known as the exponential function and ***Equation 1*** is an exponential equation. The graph of a typical exponential function is shown above. It assumes we started with 100 items and shows the expectation values at various times, t. The first two half-lives ($t_{½}$ and 2× $t_{½}$) and the lifetime, τ, are indicated on this graph. Note that $t_{½}$ is less than **τ.**

One of the many interesting facts about the exponential function is this; it does not matter when you start counting. Call the value at that moment the initial value. The time constant you measure is the same no matter where on the curve you start counting. The time constant will always be the same number of seconds, provided the system is undergoing an exponential decay process, of course.

The rate constant, the lifetime, and the half-life of the process are related. If you know one then you automatically know the others (*after a little calculation*), because $t_{½} = (\ln 2) \times \tau = (\ln 2) / \lambda$.

Half-Life vs Fraction Remaining - There is another relationship between the number of items remaining and the half-life. It derives from the definition of the logarithm and gives the fraction remaining, $N(t)/N_0$, after time t has elapsed,

$$N(t)/N_0 = \boldsymbol{e}\hat{}\{-\lambda t\} = \boldsymbol{e}\hat{}\{-[(\ln 2)/\, t_{1/2}]\, t\} = [\boldsymbol{e}\hat{}\{-(\ln 2)\}]\hat{}\{t/\, t_{1/2}\} = (1/2)\hat{}\{t/\, t_{1/2}\}$$

or

$$N(t)/N_0 = (1/2)\hat{}\{n\} = \tfrac{1}{2}^{n}, \qquad \textit{Equation 2}$$

thus

$$n = \ln[N(t)/N_0] / \ln(\tfrac{1}{2})$$

Where **n** is the number of half-lives that have elapsed, $(t/\, t_{1/2})$. The number of half-lives need not be an integer.

In nuclear experiments, we often use a graph of the activity (*in Geiger counter counts per second*) to determine the half-life. Activity is easier to measure than the number of atoms, and both can be used to determine the half-life, lifetime, and rate constant of the nuclear decay process under study. In terms of this discussion, activity is simply the number of atoms that decay during each unit time interval.

In this lab you will collect data on the number remaining. From that data you will be able to calculate the activity and plot it for comparison.

Linear Form of the Decay Curve - The exponential equation can also be written in a form that graphs as a straight line by taking the natural logarithm of both sides of the equation.

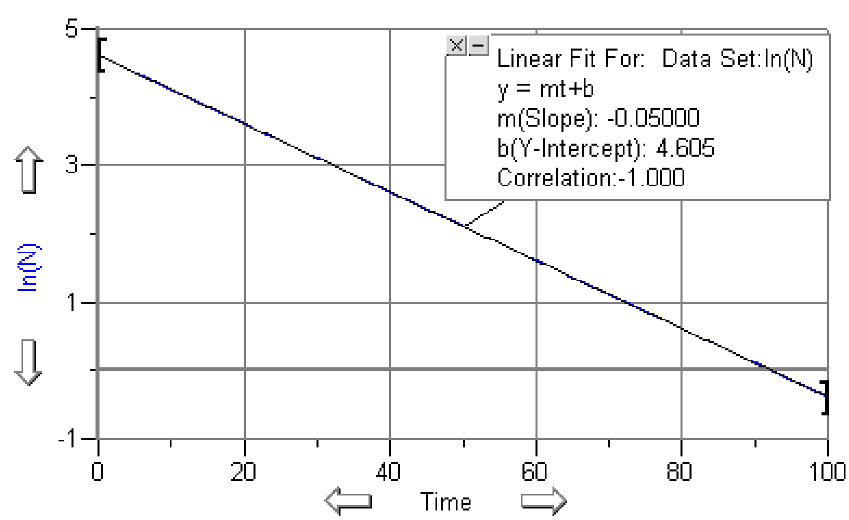
$$\ln N = \ln N_o - \lambda t = \ln N_o - \mathbf{k}t = \ln N_o - t/\tau$$

This Figure shows the graph of the equation

$$\ln N = \ln 100 - \lambda * t = \ln 100 - k * t = \ln 100 - t / \tau$$

$$\ln N = 4.605 - 0.05000 * t = 4.605 - t / 20.00$$

In a graph of the logarithmic form of the exponential decay equation, the slope of the line is $-\lambda$, $-\mathbf{k}$, or $-1/\tau$. For this type of graph, too, we can substitute activity for the number of nuclei and find the same values of slope, λ, **k** and τ from the slope.



Plotting the natural logarithm of the activity produces a line with the same slope as a plot of the natural logarithm of the number of nuclei remaining. Since both lines should have the same slope, they should provide us with identical information about the half-life, lifetime, and rate constant.

The interpretation of half-life is that it is the time required for the number of nuclei, or the activity, to fall to one-half its initial value.

The interpretation of τ is that it is the time required for the number of nuclei, or the activity, to fall to one-over-$\boldsymbol{e}$ $(1/\boldsymbol{e})$ of its initial value.

Name: ______________________________ **Period:** _______ **Due Date:** ____________

EXPONENTIAL DECAY – H4

Purpose: Study simple physical systems that are analogs of a wide variety of natural systems: systems like; spontaneous nuclear decay; voltage, current or charge on a discharging capacitor; atmospheric pressure as a function of altitude, and many others. Many physical, chemical, biological and ecological systems exhibit time-dependent behavior that can be modeled using the exponential decay function. We will use the same mathematical expression to model the four simple, mechanical systems studied in this lab.

All these systems are subject to the same probability rules: The probability of a specified decay event is the same for all particles; the probability of decay does not change with time; and, therefore, the instantaneous rate of decay (*the activity*) depends solely on the number of particles present at that moment.

Our Mechanical Model-Systems

We will simulate nuclear decay, in particular, and observe exponential decay, in general, by rolling dice, tossing two sizes of uni-connector blocks, and by flipping coins. We define dice decay by removing any die that shows six spots on top at the end of each roll. We define decay for uni-connector blocks by removing any block with its connector pointing upward at the end of each toss. We define decay for pennies by removing any penny lying "heads up" at the end of each flip. All these events have a fixed probability of outcome that doesn't change from roll to roll, from toss to toss, or from flip to flip. You will graph the surviving population count and the activity vs the number or rolls, tosses or flips (*R/T/F*). You will also plot the logarithms of these values. You will use all these graphs to determine the rate constants, lifetimes, and half-lives of these events.

The dice and the coins both have well defined and well understood probabilities. We can predict the outcomes in both these cases. The two sizes of uni-connector blocks differ in shape, size and mass. You must determine the probabilities experimentally. It remains to be seen if we can come up with a rationale for the measured probabilities. The behavior of uni-connector blocks continues to be an area of active research. We hope to understand why we get the measured probabilities and how they depend on the properties of the blocks.

You will first construct four plots of the number surviving objects vs R/T/F (*simulating the time*). Then you will graph the natural logarithm of the number. You will also create two more graphs of activity vs R/T/F and the natural logarithm of the activity. All four graphs will allow you to estimate the rate constant, lifetime, half-life, and survival and decay probabilities of each system. These four graphs will confirm that both population and activity graphs yield consistent results.

Note that short half-lives, large rate constants, and short time constants all indicate a high probability of decay events and low probability of survival. Long half-lives, small rate constants, and long time constants all indicate a low probability of decay events and a high probability of survival.

You can begin by transferring the All-Classes data for the number remaining after each roll/toss/flip into the following four tables. The instructor will then explain how to fill-in the remaining blanks in these tables. Wait for instructions before you try to proceed.

Procedure: DICE

1. Count out 100 dice (*10 x 10 array in the box*). Roll them into a working area on the lab floor. Remove the dice showing six spots. Record the number remaining.
2. Roll only the remaining dice, remove the dice showing 6 spots. Record the number remaining.
3. Repeat step #2 eighteen more times, or until there are no dice left.

DATA TABLE: DICE

(Note: 6 place logarithms must be included here unless they appear in the printed data tables with your graphs.)

	Dice Remaining after each roll		*Dice Removed during each roll*	
Roll #	*N-All Groups*	*ln N*	*A = Activity All Groups*	*ln A*
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

Procedure: Small ASYMMETRIC BLOCKS (*Use 1 cm; uni-connector blocks*)

1. Count out 100 small blocks (*each bag should contain 100 blocks*). Toss them into a working area on the lab floor. Remove the blocks that land with their connectors pointing up. Record the number remaining.
2. Toss only the remaining blocks. Remove blocks that land connector up. Record the number remaining.
3. Repeat step #2 eighteen more times, or until no blocks remain.

DATA TABLE: S-BLOCKS

(Note: 6 place logarithms must be included here unless they appear in the printed data tables with your graphs.)

	S-Blocks Remaining after each toss		S-Blocks Removed during each toss .	
Toss #	N-All Groups	ln N	A = Activity All Groups	ln A
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

Procedure: Large ASYMMETRIC BLOCKS (*Use 2 cm; uni-connector blocks*)

1. Count out 100 large blocks (*10 stacks of 10 blocks in the box*). Toss them into a working area on the lab floor. Remove the blocks that land with their connector pointing up. Record the number remaining.
2. Toss only the remaining blocks. Remove blocks that land connector up. Record the number remaining.
3. Repeat step #2 eighteen more times, or until no blocks remain.

DATA TABLE: L-BLOCKS

(Note: 6 place logarithms must be included here unless they appear in the printed data tables with your graphs.)

	L-Blocks Remaining after each toss		L-Blocks Removed during each toss .	
Toss #	N-All Groups	ln N	A = Activity All Groups	ln A
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

Procedure: COINS

1. Count out 100 pennies. There should be 100 in the small box. Flip them in groups of 5 or so into a large collection area. Remove the pennies that land heads up. Record the number remaining.
2. Flip only the remaining coins in groups of 5. Remove the heads and record the number remaining.
3. Repeat step #2 eighteen more times, or until there are no coins left.

DATA TABLE: COINS

(Note: 6 place logarithms must be included here unless they appear in the printed data tables with your graphs.)

	Coins Remaining after each flip		Coins Removed during each flip	
Flip #	*N-All Groups*	*ln N*	*A = Activity All Groups*	*ln A*
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

Analysis: Overview – read this before creating your graphs.

The 100 items studied by each group is too small to provide a reliable statistical analysis. At the end of the data collection, each group turned-in its results to the instructor. The instructor added the results from all groups together to get one grand data set for each student to personally analyze. The initial values will be 100 times the number of groups. For example, if we had data from 20 groups, then the starting count for the system would be 2,000 items. If we had 140 groups, the starting count would be 14,000.

The discussion below assumes the count is N_0, but you will know the starting count, because it is the number remaining after zero R/T/F. Four Graph One you will incorporate the known value of N_0 into the equation. For Graph Two you will incorporate the value of Ln N0 into the equation used to fit the data. used to fit the data. Make sure you use the correct starting value and the logarithm of the correct starting value in the appropriate places.

You will be making four graphs.

Graph One – Number remaining vs R/T/F
Graph Two – Ln (*Number remaining*) vs R/T/F
Graph Three – Activity vs R/T/F
Graph Four – Ln (*Activity*) vs R/T/F

Each graph must display results for all four objects on one page. Each set of results must be fit with the appropriate equation to determine the estimates of the various rate constants; λ_{Dice}, $\lambda_{S\text{-}Blocks}$, $\lambda_{L\text{-}Blocks}$, $\lambda_{Coins.}$

All of the experimental data must be duplicated and all the calculations and graphs must be created in one ***LoggerPro 3*** file. Use the Page menu to create separate pages for the four graphs and a fifth page to display a copy of the large Data Table. Be sure to save this file often while you work on it. DO NOT create any new ***LoggerPro 3*** data sets. All the data should be in a single large data table so that each graph can access all of the data. Create the graphs in the order described and follow the instructions carefully.

You will need a total of nine manual columns (*one column for R/T/F, plus four columns for Number Remaining, plus four more columns for the activity*) and eight calculated columns (*four columns to calculate the natural logarithm of the number remaining plus four more columns to calculate the natural logarithm of the activity*). Create the columns in the order described. After all the graphs are completed, create a fifth page. Copy the previous page. On this final page you must delete the graph window and expand the data table to show all the columns. Change the contents of the text box on page 5 to include your name, the lab number, and the words "Data Table." Adjust the widths of the columns, if necessary, so that all the data in all the columns is visible on this last page. Save file one more time before printing the graphs and the data table.

Make your own graphs. Do not rely on anyone else to do this work for you. Sharing graphs is not allowed.

Create the graphs so that each displays the appropriate data for all four systems on the same graph.

Name: ______________________________________

GRAPH ONE – Page 1 of ***LP3***

1) Rename the X-column Roll/Toss/Flip (*R/T/F for short*) and manually enter R/T/F numbers from 0 to 20.

2) Rename the Y-column **Dice#** (**D#** *for short*) and enter the number of dice remaining after each roll. Of course, the original quantity is the # of Dice remaining after zero rolls. This is the number at the top of the *All Classes* column.

3) **Click on Data, then New Manual Column.**
Title the third column **S-Block#** (**S-B#** *for short*) and enter the number of small blocks remaining, starting with the initial number remaining after zero tosses.

4) **Click on Data, then New Manual Column.**
Title the fourth column **L-Block#** (**L-B#** *for short*) and enter the number of large blocks remaining, starting with the initial number remaining after zero tosses.

5) **Click on Data, then New Manual Column.**
Title the fifth column **Coin#** (**C#** *for short*) and enter the number of coins remaining, starting with the initial number remaining after zero flips.

Left click on the vertical-axis label in the graph window. At the bottom of the list click on More…. Put an X in boxes for Dice#, S-Block#, L-Block#, and Coin#. Click on the horizontal-axis label in the graph window. Select Roll/Toss/Flip number from the pull-down menu. In the Graph Option menu de-select "connect the points" and select "point protectors." Then, select "Autoscale From 0" for both axes. (*Tip: after following these instructions increase the horizontal scale to just above 20. Finally, scale the vertical axis to be slightly larger than the largest N_0. These adjustments don't materially change the graph, they make it easier to see the data points.*)

In the following section you will fit the four data sets to the exponential decay function. All four graphs and all four fits will be on the same page when you print this graph. Try to prevent the data being covered up or obscured by the boxes with the fitted parameters. Likewise, make sure all the boxes are contained on the page so that you and the instructor can read you fitting parameters on the printed graph.

Please note that N_0 and $\ln(N_0)$ are not fitted parameters. You will type these numbers into the equations rather than letting ***LoggerPro 3*** find them for you. On the other hand, A_0 and $\ln(A_0)$ are fitted parameters. (*The numeric values λ written on the next page must agree with your graphical fits.*)

Name: ____

GRAPH ONE - *continued*

DICE#: Perform a fit on the DICE curve.

Use the *curve fit* menu option to find the best fit for this DICE line. Start with the Natural Exponent function, A*exp(−C*x) + B. Use the **Define Function** button and delete the +B. Then change the upper case C to an upper case B. Also, change A to the known value of N_o. The new function will appear at the end of the list and should look something like this: y = 15500*exp(−B*x) (***assuming N_o equals 15,500, if it's not 15,500, use the correct number, the correct number is the one at the top of the data column***). Do not let ***LP3*** search for the value of N_o; plug the known numerical value of N_0 into the equation for these graphic fits.

Record the function parameters from the best fit to the DICE line: (*Number here must agree with your graph*)

A = N_o = _____________;   B = λ_{Dice} = _______________ $roll^{-1}$

S-BLOCK#: Perform a fit on the S-BLOCKS curve.

Define a similar function to find the best fit for the BLOCKS line (*only the N_0 value will be different*).

Record the function parameters from the best fit to the block data: (*Number here must agree with your graph*)

A = N_o = _____________;   B = λ_{Blocks} = _______________ $toss^{-1}$

L-BLOCK#: Perform a fit on the L-BLOCKS curve.

Define a similar function to find the best fit for the BLOCKS line (*only the N_0 value will be different*). .

Record the function parameters from the best fit to the block data: (*Number here must agree with your graph*)

A = N_o = _____________;   B = λ_{Blocks} = _______________ $toss^{-1}$

COIN#: Perform a fit on the COINS curve.

Define a similar function to find the best fit for the COINS line (*only the N_0 value will be different*). .

Record the function parameters from the best fit to the coin data: (*Number here must agree with your graph*)

A = N_o = _____________;   B = λ_{Coins} = _______________ $flip^{-1}$

In the Graph Option menu select "Autoscale From 0" for both axes.

Save the File one more time before you do anything else.

Name: ____

GRAPH TWO – Page 2 of ***LP3***

Use the page menu to add a new page. At the screen, select the radio button to "copy current page". This will carry all the data over to the new page. DO NOT create a new data set. All the data must be in one large data table so it will all be available to all four of your graphs. By putting each graph on its own page, you preserve it in case you need to make a revision later.

Add four calculated columns (*6 thru 9*) for the **natural logarithms** of the Dice#, the S-Block#, the L-Block#, and the Coin#. Edit the graph on page 2 to show ***ln* N** (*vertical axis*) **vs R/T/F number**. These should give four straight lines. Use the Curve Fit menu option to find the best *linear* fit (start with y = m*x + b and then define the function) for each of these lines. Use the **Define Function** button to set b equal to **ln** N_o.
For example, if N_o = 15,500, then

Define the Function as m*x+ln(15500)

OR, you could calculated **ln** N_o and enter that value directly; for example **ln** 15500 = 9.648595303

Define the Function as m*x+9.648595303

Do not let ***LP3*** search for **ln** N_o; set it to the known value for yourself. The slope will be $-\lambda$.
(*The numeric values of* λ *written in the blanks below must agree with your graphical fits.*)

Dice#: b = ln N_o = ____________________; N_o = ____________________;

m = slope = ____________________ = $-\lambda$, so λ_{Dice} = ____________________ $roll^{-1}$

S-Block#: b = ln N_o = ____________________; N_o = ____________________;

m = slope = ____________________ = $-\lambda$, so $\lambda_{SBlocks}$ = ____________________ $toss^{-1}$

L-Block#: b = ln N_o = ____________________; N_o = ____________________;

m = slope = ____________________ = $-\lambda$, so $\lambda_{LBlocks}$ = ____________________ $toss^{-1}$

Coin#: b = ln N_o = ____________________; N_o = ____________________;

m = slope = ____________________ = $-\lambda$, so λ_{Coins} = ____________________ $flip^{-1}$

In the Graph Option menu select "Autoscale From 0" for both axes.

Save the File one more time before you do anything else.

Name: ______________________________________

GRAPH THREE – *Page 3 of* ***LP3***

Add four more columns (*10 thru 13*) for the **activity** of the A-Dice, the A-S-Blocks, the A-L-Blocks, and the A-Coins. Enter the activities starting at the row with **R/T/F number** = 1. The top row should be left blank. Create a graph of **activity** (*vertical axis*) **vs R/T/F number**. These should give four exponential curves. Start with the Natural Exponent function, then use the **Define Function** button to remove the +B. Change the C to a capitol B. The new function will appear at the end of the list and should look something like this:

$$y = A_0*\exp(-B*x).$$

Let ***LP3*** search for the initial value of the activity, A_0, and the negative value of the rate constant. (*The numeric values of* A_0 *and* λ *written on the next page must agree with your graphical fits.*)

Start each new column by clicking on **Data; New Manual Column.**

Title the new column **A-Dice** (**A-D** *for short*) and manually enter the number of dice lost between rolls.

Title the next column **A-S-Blocks** (**A-S-B** *for short*) and manually enter the number of small blocks lost during each toss.

Title the next column **A-L-Blocks** (**A-L-B** *for short*) and manually enter the number of large blocks lost during each toss.

Title the next column **A-Coins** (**A-C** *for short*) and manually enter the number of coins lost during each flip.

Click on the y-axis of the graph window. Put an X in boxes for A-Dice, A-S-Blocks, A-L-Blocks, and A-Coins.

Click on the x-axis of the graph window. Select roll/toss/flip number from the pull-down menu.

In the Graph Option menu select "Autoscale From 0" for both axes.

Name: ____

GRAPH THREE *- continued*

A-Dice: **Perform a fit on the Activity of Dice curve.**

Use the Curve Fit menu to find the best fit for this Activity of Dice curve. Start with the Natural Exponent function, then use the **Define Function** button to remove the +B. Change the C to a B. The new function will appear at the end of the list and should look something like this: y = A*exp(–B*x) (*Let **LP3** find the initial activity, A_0*).

Record the function parameters from the best fit to the A-Dice# curve.

A_o = ____________; B = λ_{Dice} = ______________ $roll^{-1}$

A-S-Blocks: **Perform a fit on the Activity of Small Blocks curve.**

Use the same function to find the best fit for the Activity of Small Blocks curve.

Record the function parameters from the best fit to the block data:

A_o = ____________; B = λ_{Blocks} = ______________ $toss^{-1}$

A-L-Blocks: **Perform a fit on the Activity of Large Blocks curve.**

Use the same function to find the best fit for the Activity of Large Blocks curve.

Record the function parameters from the best fit to the block data:

A_o = ____________; B = λ_{Blocks} = ______________ $toss^{-1}$

A-Coins: **Perform a fit on the Activity of COINS curve.**

Use the same function to find the best fit for the Activity of Coins curve.

Record the function parameters from the best fit to the coin data:

A_o = ____________; B = λ_{Coins} = ______________ $flip^{-1}$

In the Graph Option menu select "Autoscale From 0" for both axes.

Save the File one more time before you do anything else.

Name: ____

GRAPH FOUR – *Page 4 of* ***LP3***

Add four more columns (*14 thru 17*) for the **natural logarithms** of the A-Dice, the A-S-Blocks, the A-L-Blocks, and the A-Coins. Create a graph of ***ln*** **A** (*vertical axis*) **vs R/T/F number**. These should give four straight lines. Use the Curve Fit menu option to find the best *linear* fit (y = m*x + b) for each of these lines. (*Let* ***LP3*** *find the best value of* $\boldsymbol{b = \ln A_0}$*, this time.*) The slope will be $m = -\lambda$.

Let ***LP3*** calculate the logarithm of the initial activity.
(*The numeric values of* $\ln A_0$ *and* λ *written in the blanks below must agree with your graphical fits.*)

A-Dice: $b = \ln A_o =$ ____________________; $A_o =$ ____________________;

$m = \text{slope} =$ ____________________ $= -\lambda$, so $\lambda_{Dice} =$ ____________________ roll^{-1}

A-S-Block: $b = \ln A_o =$ ____________________; $A_o =$ ____________________;

$m = \text{slope} =$ ____________________ $= -\lambda$, so $\lambda_{SBlocks} =$ ____________________ toss^{-1}

A-L-Block: $b = \ln A_o =$ ____________________; $A_o =$ ____________________;

$m = \text{slope} =$ ____________________ $= -\lambda$, so $\lambda_{LBlocks} =$ ____________________ toss^{-1}

A-Coin: $b = \ln A_o =$ ____________________; $A_o =$ ____________________;

$m = \text{slope} =$ ____________________ $= -\lambda$, so $\lambda_{Coins} =$ ____________________ flip^{-1}

In the Graph Option menu select “Autoscale From 0” for both axes.

Save the File one more time before you do anything else.

Name: ______________________________________

Calculating the Half-life

The Half-life, $t_{½}$, is the time needed for half of the dice, blocks, or coins to be removed. λ and $t_{½}$ are related;

$$\lambda = (ln\ 2) / t_{½} = 0.693 / t_{½} \text{ therefore } t_{½} = (ln\ 2) / \lambda = 0.693 / \lambda \qquad \textit{Equation 3}$$

Dice:

1) $t_{½\_Dice}$ (*from Graph 1*) = __________________ rolls
2) $t_{½\_Dice}$ (*from Graph 2*) = __________________ rolls
3) $t_{½\_Dice}$ (*from Graph 3*) = __________________ rolls
4) $t_{½\_Dice}$ (*from Graph 4*) = __________________ rolls

S-Blocks:

1) $t_{½\_SBlocks}$ (*from Graph 1*) = __________________ tosses
2) $t_{½\_SBlocks}$ (*from Graph 2*) = __________________ tosses
3) $t_{½\_SBlocks}$ (*from Graph 3*) = __________________ tosses
4) $t_{½\_SBlocks}$ (*from Graph 4*) = __________________ tosses

L-Blocks:

1) $t_{½\_LBlocks}$ (*from Graph 1*) = __________________ tosses
2) $t_{½\_LBlocks}$ (*from Graph 2*) = __________________tosses
2) $t_{½\_LBlocks}$ (*from Graph 3*) = __________________ tosses
4) $t_{½\_LBlocks}$ (*from Graph 4*) = __________________ tosses

Coins:

1) $t_{½\_Coins}$ (*from Graph 1*) = __________________ flips
2) $t_{½\_Coins}$ (*from Graph 2*) = __________________ flips
3) $t_{½\_Coins}$ (*from Graph 3*) = __________________ flips
4) $t_{½\_Coins}$ (*from Graph 4*) = __________________ flips

The half-lives derived from the exponential and linear fits should be very close to these predictions for the Dice and Coins. If they are not very similar, then you have a math error somewhere in your graph or your calculations. If so, find it, fix it and recalculate.

There is a separate handout where we can discuss the probabilities for the large and small blocks. Neither of these has an obvious expectation value for the probability of survival.

Note the alternative method of calculating the half-life introduced in this section.

<u>What do we think the λ's, τ 's, and $t_{1/2}$'s of the Coin and Dice systems should be?</u>

Coins

For the coins, that's an easy question. The probability of not tossing heads and surviving to next round is $\langle P_{\text{not Heads}} \rangle = 0.5000$. The probability of tossing heads and not surviving to the next round is $\langle 1 - P_{\text{not Heads}} \rangle = 0.5000$. We expect to lose half and retain half of the coins on each toss. This suggests that we can expect the half-life in the coin tossing section of the lab to be

$$t_{1/2} = 1.000 \text{ flip of the coins,}$$

Therefore,

$$\lambda = \ln(2)/t_{1/2} = 0.693147/1.000 = 0.693147 \text{ flip}^{-1},$$

and thus, the lifetime

$$\tau = 1/\lambda = 1.44270 \text{ flips}$$

Compare these expectations to your results. It should be instructive. If your results are not close to these expectation values, then make an effort to find your error.

Dice

For the dice, the probability of not rolling a six and surviving to the next round is expected to be, $\langle P_{\text{not Six}} \rangle = 5/6 = 0.8333333$. The probability of rolling a six and not surviving to the next round is therefore expected to be, $\langle 1 - P_{\text{not Six}} \rangle = 1/6 = 0.1666666$. Then,

$$\lambda = -\ln \langle P_{\text{not Six}} \rangle = -\ln(5/6) = 0.182322 \text{ roll}^{-1}.$$

The half-life is therefore,

$$t_{1/2} = \ln(2)/\lambda = 3.80178 \text{ rolls.}$$

And thus the lifetime

$$\tau = 1/\lambda = 5.48481 \text{ rolls}$$

Working with Half-Lives

There is an alternative route to finding the half-life. Raise the <u>survival probability</u> to the unknown exponent, **r**, and set it equal to ½ = 0.500000. The unknown exponent is the "number of rolls, tosses or flips" needed to reach the point where only half of the original population remains. By definition then, **r** is the half-life = number of rolls, tosses or flips needed to eliminate half of the original population. Compare an example with the results above.

For example, try this calculation for the dice. To find the half-life we need to find the exponent, **r**, in this equation

$$\langle P_{\text{not Six}} \rangle^{r} = (0.833333)^{r} = 0.50000000$$

Take the logarithm and solve

$$\mathbf{r} = \ln(0.500000)/\ln(0.833333) = t_{1/2}$$

$$t_{1/2} = 3.80178 \text{ rolls of the dice.}$$

Therefore

$$\lambda = \ln(2)/\, t_{1/2} = 0.182322 \text{ roll}^{-1}.$$

Therefore

$$\tau = 1/\lambda = 5.48481 \text{ rolls}$$

Name: ____________________________________

λ *and Probability*

The survival probability, $\mathbf{P_{survival}}$, of any member of the set avoiding elimination and surviving the next unit time interval is, strictly as a matter of definition,

$$\mathbf{P_{survival}} = \frac{\mathbf{N}(at\ time\ t+1)}{\mathbf{N}(at\ time\ t)} = \frac{N_0 e^{-\lambda(t+1)}}{N_0 e^{-\lambda t}} = e^{-\lambda t-\lambda-(-\lambda t)} = e^{-\lambda}; \text{ where } \mathbf{0} < \mathbf{P} < \mathbf{1}$$

From this we derive the relationships between **P** and λ, and between τ, and $t_{½}$,

$$\lambda = -\ln(\mathbf{P_{survival}}) = 1/\tau = (\ln 2) / t_{½} \qquad \textit{Equation 4}$$

$$\tau \ln(2) = t_{½} \qquad \textit{Equation 4a}$$

The sum of the probabilities of all the possible outcomes in one unit time interval add up to one. There are only two possible outcomes; either a specific member of the set survives or it does not. The probability of any member of the set being eliminated, i.e. failing to survive the next unit time interval, must be ‹$\mathbf{1 - P_{survival}}$›.

‹$\mathbf{P_{survival}}$› and ‹$\mathbf{1 - P_{survival}}$› add up to one, as required, and each must be constant for systems that exhibit exponential decay.

In our systems we use rolls, tosses, and flips as proxies for the unit time interval. Each roll, toss, or flip is treated as one unit of time. Our λ therefore, has units of $toss^{-1}$, $roll^{-1}$, or $flip^{-1}$. When **P** is constant, so is λ.

Equation Summary – for quick reference

$$N(t) = N_o e^{-\lambda t} = N_o e^{-\mathbf{k}t} = N_o e^{-t/\tau} \qquad (\textit{Exponential Decay Equation}) \qquad \textit{Equation 1}$$

$(\tau = 1/\mathbf{k} = 1/\tilde{\lambda})$.

$t_{½} = (ln\ 2) \times \tau = (ln\ 2) / \lambda$.

$N(t)/N_0 = e$^{$-\lambda t$} = (1/2)^{n} = $½^n$, where $n = t/t_{½}$, thus

$n = \ln[N(t)/N_0] / \ln(½) =$ *number of half-lives that have passed* **Equation 2**

$$ln\ N = ln\ N_o - \lambda t = ln\ N_o - \mathbf{k}t = ln\ N_o - t/\tau \qquad (\textit{Linear form of Exponential Decay Equation})$$

$$\lambda = (ln\ 2) / t_{½} = 0.693 / t_{½} \quad \text{therefore} \quad t_{½} = (ln\ 2) / \lambda = 0.693 / \lambda \qquad \textit{Equation 3}$$

$$\mathbf{P_{survival}} = \frac{N(at\ time\ t+1)}{N(at\ time\ t)} = \frac{N_0 e^{-\lambda(t+1)}}{N_0 e^{-\lambda t}} = e^{-\lambda t-\lambda-(-\lambda t)} = e^{-\lambda}; \text{ where } \mathbf{0} < \mathbf{P} < 1$$

From these we derive the relationship between **P** and λ, τ, and $t_{½}$,

$$\lambda = -\ln(P) = 1/\tau = (\ln 2) / t_{½} \quad \text{and} \quad \tau \ln(2) = t_{½} \qquad \textit{Equation 4}$$

Name: ____

λ *and Probability* *- continued*

Your graphs provide you with four estimates of λ for each system. Use the average λ to calculate both 1−P, the probability of elimination events occurring (*six, connector up, or heads*), and P, the probability of not being eliminated (*not a six; not connector up; or not heads*) in each interval. Also use the average λ to calculate the average half-life and average lifetime of each system.

Average λ_{Dice} (*for eliminating dice*) = _____________; **‹$P_{not\ six}$›** =_____________; **‹1− $P_{not\ six}$›** = ______________

You expect the probability of not rolling a six (*surviving the event*) = **‹$P_{not\ six}$›** =__________________________;

Average Dice: $t_{½\_Dice}$ = ________________________ ; τ_{Dice} = ________________________

Average $\lambda_{SBlocks}$ (*for eliminating S-Blocks*) = __________; **‹$P_{not\ up}$›** = ___________; **‹1− $P_{not\ up}$›** = __________

You expect the probability of not tossing a connector up (*surviving the event*) = **‹$P_{not\ up}$›** = **0.833 or higher**

Average S-Blocks: $t_{½\_S\text{-}Block}$ = ________________________ ; $\tau_{S\text{-}Block}$ = ________________________

Average $\lambda_{LBlocks}$ (*for eliminating L-Blocks*) = __________; **‹$P_{not\ up}$›** = __________; **‹1− $P_{not\ up}$›** = ___________

You expect the probability of not tossing a connector up (*surviving the event*) = **‹$P_{not\ up}$›** = **0.7500-0.8333**
.

Average L-Blocks: $t_{½\_L\text{-}Block}$ = ________________________ ; $\tau_{L\text{-}Block}$ = ________________________

Average λ_{Coins} (*for eliminating coins*) = _____________, **‹P_{tails}›** = _____________, **‹1− P_{tails}›** = _____________

You expect the probability of not flipping heads (*surviving the event*) = **‹P_{tails}›** = __________________________.

Average Coin: $t_{½\_Coin}$ = ________________________ ; τ_{Coin} = ________________________

Name: ____

Questions for analysis: *(The questions are on WebAssign. These examples are for practice only.)*

1. If the half-life of U-238 is 4,500,000,000 years, what is the rate constant of decay, λ?
   What are the units of your λ?

2. If the half-life of carbon-14 is 5720 years, how many <u>half-lives</u> and <u>years</u> will pass before only 1/100th of the original amount remains.

3. Why, as a practical matter, is C-14 dating not used to date objects older than about 50,000 years?

4. Why can't C-14 be used to date a 2,500-year-old clay pot found in a cave in Greece? (*The clay used to make the pot usually contains some amount of organic carbon. Even so, it cannot be used to date the pot. Why not?*)

5. The half-life of Indium-116 (^{116}In) is 54 minutes.
   a) What is the decay rate constant λ (*in units of* min^{-1}).

   b) What fraction of a sample of ^{116}In remains after it is allowed to decay for 80 minutes?

6. The activity of a radioactive element is the number of decays per second a sample produces. This activity is related to the decay rate constant by -- Activity = λ N -- where N is the number of atoms present at any given moment. Suppose that you have 2.5 nanograms of radon gas in a room. The half-life of radon gas is 3.82 days.
   a) Use Avagadro's number and the atomic mass of radon to find the number of atoms of radon initially present in the room.

   b) Use the half-life to determine the decay rate constant (λ, *in units of* s^{-1}) for radon.

   c) Determine the initial activity of the radon in the room in decays per second.

   d) What mass of radon will remain in the room after 14 days?

7. <u>If</u> the Shroud of Turin has a C-14 radioactivity decay rate of 4.7 decays/sec and your shirt has a rate of 5.0 decays/sec, how old is the Shroud? (*Hint:* N/N_o *can be replaced with* $Activity/Activity_o$.)

Name: ______________________________ **Period:** ________

EXPONENTIAL DECAY – H5

A Possible Explanation for the Large and Small Block Survival Probabilities

To estimate the probabilities for the block systems, we begin by imagining that the block is entering its final quarter of a roll. Each die can start its final quarter turn from one of six sides. If a block rolls from the bottom side, a quarter roll takes it to one of the <u>four adjacent sides</u>. That is 4 possible outcomes for each starting position.

Therefore, 24 distinct outcomes (*six possible last-roll faces × four possible landing faces for each*) are possible.

If it starts on the <u>bottom face</u>, a quarter roll takes it onto one of the four <u>side faces</u>. (*4 possible outcomes.*)

If it starts <u>on the connector</u>, a quarter roll always takes it to one of the four <u>side faces</u>. (*4 possible outcomes.*)

If it lands on one of <u>the side faces</u> (*there are four side faces*),

there are two chances of rolling onto another of the <u>side faces</u>, (*That is 4 × 2 = 8 possible outcomes*.)

one chance of rolling onto the bottom with the <u>connector up</u>, (*That is 4 × 1 = 4 possible outcomes*.)

and one chance of rolling onto the connector with <u>connector down</u> (*That is 4 × 1 = 4 possible outcomes*.)

In summery, the 24 possible outcomes are expected to produce:

4 with a connector up,
4 with a connector down, and
16 resting on one of the side faces (*with a side face up, of course*).

If it were that simple, the probability of tossing a block with its connector up would be 4 out of 24, which is one-in-six; the same probability as rolling a six on the dice. The expectation value for "not connector up" would be $\langle P_{\text{not up}} \rangle = 5/6 = 0.833333333$. This disagrees with your data. It is not a good enough model to explain what we observe.

To improve the model, we need to focus on those four chances out of 24 that should have landed on the connector (*that is, the <u>connector down</u> outcomes*). Experience suggests that large blocks hardly ever land with their connectors down, while the small blocks often do.

It is this observed difference in the <u>connector down</u> outcomes that suggests the changes in the model discussed on the next page.

Modifying the Model for Large and Small Blocks

For a large block, a roll onto the connector may have, probably does have, enough energy to keep it rolling. In that case, we will get fewer than four landing on the connector and more than 16 landing on the side. If it was that simple, however, we would still end up with $\langle P_{not\ up} \rangle = 5/6 = 0.833333333$, since landing on the side is as good as landing on the connector. The block survives in either case. This is still not a good enough model to explain what we observed.

It seems that we are forced to imagine that if the large block lands on its connector, it sometimes has enough potential energy to roll for two additional quarter-turns. In this model, tosses that should land on the connector sometimes roll one quarter-turn and sometimes roll an two quarter-turns. In the latter even the block lands with the connector pointing up.

When a block lands on the connector, the added potential energy it contains from the increase in height might be sufficient to allow it to continue rolling; sometimes, by more than one quarter-turn. Thus, a landing that should stop on the connector will sometimes stop with the connector up. This would contribute to the observed increase in λ for large blocks relative to dice.

The fraction of connector landings that stop with the connector up does not need to be a simple fraction of the total. It depends on the geometry of the landing as well as on the energy of the rolling block. Some roll a quarter of a turn and stop on the side. Some roll two quarter-turns and stop with the connector up.

In the original model there were 4 connector-up landings. With this revised model there could be up to four more connector-up landings. There are still only 24 outcomes in the count, but for the large blocks the four connector-down landings are all converted to either side landings or to connector up landings.

In this revised model for the large blocks, the number of connector-up landings will be somewhere between four and eight, inclusive. Here is a table of the probabilities, λ's, τ 's, and $t_{1/2}$'s for each integer number of connector-up landings.

#Connector-up	$\langle 1-P \rangle_{connector\ up}$	$\langle P_{not\ up} \rangle$	λ	τ	$t_{1/2}$
4	4/24 = 0.166667	0.833333	0.182322	5.48481	3.80178
5	5/24 = 0.208333	0.791667	0.233615	4.28055	2.96705
6	6/24 = 0.250000	0.750000	0.287682	3.47606	2.40942
7	7/24 = 0.291667	0.708333	0.344840	2.89989	2.01005
8	8/24 = 0.333333	0.666667	0.405465	2.46630	1.70951

Use the experimental probability measured for the large blocks to interpolate from the values in this table the expected number of connector-up landings out of twenty-four.

Number of connector-up landings out of 24 = ______________ / 24

This model and these calculations apply to the large blocks.

The small blocks seem to exhibit a different behavior and require a different model. The small blocks are not only smaller than the large blocks, they are made of a different material. Furthermore, the connectors are not in the same proportion on the small and large blocks. This combination of differences leads to the following suggested model for the small blocks.

It appears that the small blocks do not have enough energy to roll up on their connectors at critical stages of a roll. The connector or the material inhibit the block from rolling further and it sits there with its connector down or it rolls backward one-quarter of a roll onto its side.

The result is that for small blocks the statistics are almost the same as for dice. The relatively small size of the connector means that it has very little effect on the outcome. It is even possible that it inhibits rolling enough to produce slightly fewer connector-up landings than expected. That would increase the probability of survival and decrease λ.

#Connector-up	**‹1 – $P_{not\ up}$›**	**‹$P_{not\ up}$›**	λ	τ	$t_{½}$
1	1/24 = 0.041666	0.958333	0.042560	23.4964	16.2865
2	2/24 = 0.083333	0.916667	0.087011	11.4928	7.96617
3	3/24 = 0.125000	0.875000	0.133531	7.48889	5.19089
4	4/24 = 0.166667	0.833333	0.182322	5.48481	3.80178

For the small blocks, we predict that the probability of surviving the next unit time interval, **‹$P_{not\ up}$›**, should be slightly larger than 5/6 = 0.833333. That means that λ should be slightly less than 0.18232.

Use the experimental probability measured for the small blocks to interpolate from the values in this table the expected number of connector-up landings out twenty-four.

Number of connector-up landings out of 24 = ______________ / 24

This model and these calculations apply to the small blocks.

Note that the ability to interpret the data for both block types in light of this model does not prove that we understand the details of the model. Nor can we prove that it is the correct model. Proof is hard to come by in scientific investigations. Often, the best we can hope for is a simple consistent model that makes sense of the data. Consistent models are usually considered reliable until unexplainable new data comes along to force a further revision.

Further Examples of Physical Systems Exhibiting Exponential Decay I.

<u>**Discharging**</u> **a Capacitor through a resistor - Theory**: A charged capacitor of capacitance $\mathbf{C} = \mathbf{Q_0}/\mathbf{V_0}$ with voltage $\mathbf{V_0}$ and stored charge $\mathbf{Q_0}$ is discharged through a resistor, **R**. There are many mobile charge carriers stored on the capacitor plates. When the **RC** circuit is shorted all the mobile charges experience an electric field. This field is created by the separation of charges within the capacitor. The motion of the charge carriers is like balls moving through a huge pinball machine.

The probability that a charge carrier makes it through this maze to a matching hole of opposite charge on the other plate depends on the number of mobile charge carriers present and on the voltage across the capacitor. Using Kirchhoff's voltage rule we can analyze how the voltage change and the change in the number of mobile charge carriers operate to produce the exponential decline observed in such systems.

Kirchhoff's rule says at each moment the sum of the voltage changes across the capacitor ($\mathbf{Q_t}/\mathbf{C}$) and the voltage drop across the resistor ($\mathbf{I_t R}$) must add up to zero. Substituting $d\mathbf{Q}/d\mathbf{t}$ for current, **I,** allows us to generate the following sequence of equations:

$$(\mathbf{Q}/\mathbf{C}) + \mathbf{R}(\mathbf{I}) = 0 \qquad \text{Kirchhoff's Voltage Rule}$$

$$\mathbf{Q}/\mathbf{C} = -\mathbf{R}\,(d\mathbf{Q}/d\mathbf{t})$$

$$-(1/\mathbf{RC})\,d\mathbf{t} = d\mathbf{Q}/\mathbf{Q}$$

Integrating these differential expressions from t_0 to t and from Q_0 to Q produces the following results for the discharging capacitor:

$$-\mathbf{t}/\mathbf{RC} = \ln \mathbf{Q_t} - \ln \mathbf{Q_0} = \ln(\mathbf{Q_t} / \mathbf{Q_0})$$

Converting to exponential form $\qquad \mathbf{Q_t} = \mathbf{Q_0}\, e^{-\mathbf{t}/\mathbf{RC}}$

And since $\mathbf{I_t} = d\mathbf{Q_t}/d\mathbf{t}$ $\qquad \mathbf{I_t} = (-\mathbf{Q_0}/\mathbf{RC})\, e^{-\mathbf{t}/\mathbf{RC}} = \mathbf{I_0}\, e^{-\mathbf{t}/\mathbf{RC}}$

And since $\mathbf{V_t} = \mathbf{I_t R}$ $\qquad \mathbf{V_t} = (-\mathbf{Q_0}/\mathbf{C})\, e^{-\mathbf{t}/\mathbf{RC}} = \mathbf{V_0}\, e^{-\mathbf{t}/\mathbf{RC}}$

These equations require a <u>fixed probability</u> that a mobile charge carrier will be neutralized in a specific unit time interval, say one second. This probability does not change with time even though the voltage driving the charges and the number of charge carriers are both changing. The changes in these two quantities balance each other exactly because Ohm's Law guarantees that their ration (V/I) must be constant.

Specifically, the time rate of change in the number of charge carriers is just the current, $\mathbf{I_t}$. The voltage at that instant is simply $\mathbf{V}_t$. The decreasing number of charge carriers should make the probability of being neutralized decrease proportionately for the remaining charge carriers. The dropping voltage should also reduce the probability of being neutralized. The ratio of these two factors cancel out producing a constant probability that a given charge carrier will be neutralized during the next unit time interval; just what the exponential decline equation is telling us (*the ratio V/I exhibits that same exponential decay function*).

The time interval, **RC** [*units: (volts / ampere)(coulombs / volt) = (coulombs / ampere) = coulombs/(coulombs/second) =seconds*], is the time it takes for the charge, current, and voltage to fall to $1/e$ of their initial values. **RC** is called the time constant, or the relaxation time, for the discharging circuit. The half-life, $t_{1/2}$, of the discharging **RC** circuit is (ln 2)×**RC**. On average, one-half of the mobile charge carriers are neutralized in one half-life, half of the remainder in the next half-life, and so forth. Graphing the number of remaining charges, the current, or the voltage versus time yields an exponential decay curve mathematically identical to that for the decay of radioactive nuclei. The rate constant, **k**, for the discharging capacitor, analogous to λ for radioactive decay, is simply 1/**RC**. The probability of a charge surviving during the next unit time interval is $\mathbf{P} = e^{-1/\mathbf{RC}}$. The probability of being neutralized in the next unit time interval is $1-\mathbf{P} = 1 - e^{-1/\mathbf{RC}}$.

Further Examples of Physical Systems Exhibiting Exponential Decay II.

The Barometer Equation for Atmospheric Pressure - Theory: This derivation assumes a dry atmosphere of an ideal gas. We wish to know the pressure at a distance **z** above the surface of the Earth. At the surface of the Earth the atmospheric pressure is $\mathbf{P_0}$. We use the symbols **R** for the universal gas constant and **m** for the molecular weight of the gas (*kg*); $\mathbf{g} = GM_E/(r_E + \mathbf{z})^2$, **T** is the absolute temperature, and ρ (*kg/m*^{3}) is the density of the air. For one mole of the gas we can rewrite the ideal gas equation as

$$\mathbf{PV} = \mathbf{RT} \qquad \mathbf{P}, \mathbf{V}, \text{ and } \mathbf{T} \text{ all vary with } \mathbf{z}.$$

$$\mathbf{P}\,(\mathbf{m}/\rho) = \mathbf{RT} \qquad \rho \text{ also varies with } \mathbf{z}$$

$$\mathbf{P} = \rho\,(\mathbf{R}/\mathbf{m})\,\mathbf{T} \qquad \mathbf{R} \text{ and } \mathbf{m} \text{ are the only constants}$$

We start with the standard hydrostatic assumption that the weight of a volume of atmosphere is almost exactly balanced by the pressure gradient. We assume the **PV**-work done by gravity lifting the gas sample to an altitude **z** equals the potential energy of the gas sample at the new altitude,

$$d(\mathbf{PV})/d\mathbf{z} = -\rho\mathbf{Vg}$$

The negative sign indicates that the pressure is dropping as **z** increases. Then, since **V** is an assumed constant for small vertical excursions, this simplifies to

$$d\mathbf{P}/d\mathbf{z} = -\rho\mathbf{g}$$

Substituting the equation of state , $\mathbf{P}\,(\mathbf{m}/\rho) = \mathbf{RT}$, into the differential equation yields

$$d\mathbf{P} = -\rho\mathbf{g}\,d\mathbf{z} = -(\mathbf{Pmg}/\mathbf{RT})\,d\mathbf{z}$$

Then

$$d\mathbf{P}/\mathbf{P} = -(\mathbf{mg}/\mathbf{R})\,(d\mathbf{z}/\mathbf{T}) \qquad (\textit{where we assume that } \mathbf{g} \textit{ is constant.})$$

Integrating the differential equation leads to the conclusion that

$$\mathbf{P} = \mathbf{P_0}\,e^{-(mg/R)\int(1/T)\,dz}$$

If we can assume that the temperature does not change with altitude (*a very rough approximation*), then we can pull temperature out of the integral and simplify the integral to **z**, the height, and obtain our pressure profile equation;

$$\mathbf{P} = \mathbf{P_0}\,e^{-(mg/RT)\,z}$$

Where **z** is the elevation above sea level. This equation predicts that the pressure drops exponentially with increasing altitude. This trend will deviate from the strict exponential trend if there are significant temperature changes over the range of altitudes being investigated.

To find the half-height, the altitude at which the pressure is one-half of the normal sea-level pressure, evaluate the equation;

$$\mathbf{z}_{1/2} = \ln(2\,\mathbf{RT}/\mathbf{mg})$$

The half-height turns out to be approximately 5.7 km (*18,700 feet*), assuming a constant **T** of 283K and **m** = 0.028 kg. By the time you get to the top of Mt. Everest (*29,028 feet, though GPS measurements suggest that it is slightly less*) the pressure is only about 1/3rd of normal atmospheric pressure.

ONE PAGE PER GROUP: This page must be turned in at the end of the lab period so that the instructor can tabulate the data from all groups

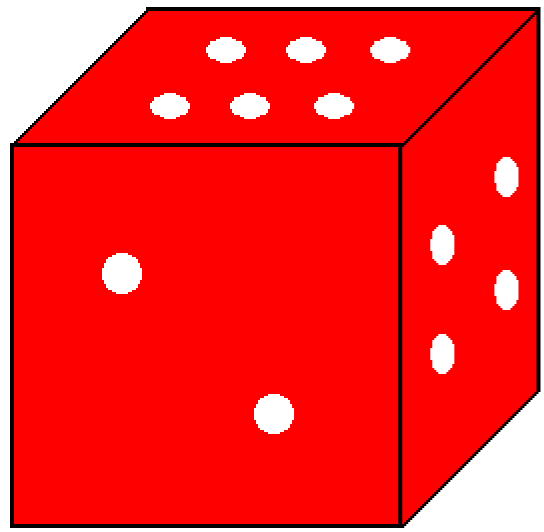
Procedure: DICE

1. Count out 100 dice (*10 x 10 array in the box*). Roll them into a working area on the lab floor. Remove the dice showing six spots. Record the number remaining.
2. Roll only the remaining dice, remove the dice showing 6 spots. Record the number remaining.
3. Repeat step #2 eighteen more times, or until there are no dice left.

DATA TABLE: DICE

Group: ____

Roll #	***Dice Remaining***
0	100
1	_____
2	_____
3	_____
4	_____
5	_____
6	_____
7	_____
8	_____
9	_____
10	_____
11	_____
12	_____
13	_____
14	_____
15	_____
16	_____
17	_____
18	_____
19	_____
20	_____



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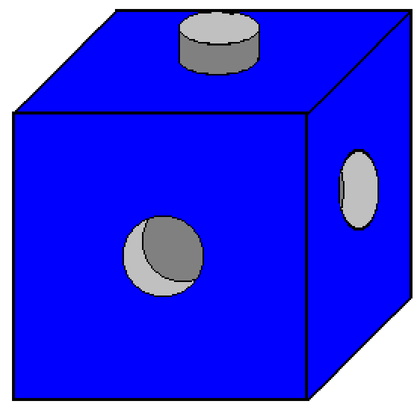
Procedure: Small ASYMMETRIC BLOCKS (*Use 1 cm; uni-connector blocks*)

1. Count out 100 small blocks (*each bag should contain 100 blocks*). Toss them into a working area on the lab floor. Remove the blocks that land with their connectors pointing up. Record the number remaining.
2. Toss only the remaining blocks. Remove blocks that land connector up. Record the number remaining.
3. Repeat step #2 eighteen more times, or until no blocks remain.

DATA TABLE: S-BLOCKS

Group: ____

Toss #	***Blocks Remaining***
0	100
1	_____
2	_____
3	_____
4	_____
5	_____
6	_____
7	_____
8	_____
9	_____
10	_____
11	_____
12	_____
13	_____
14	_____
15	_____
16	_____
17	_____
18	_____
19	_____
20	_____



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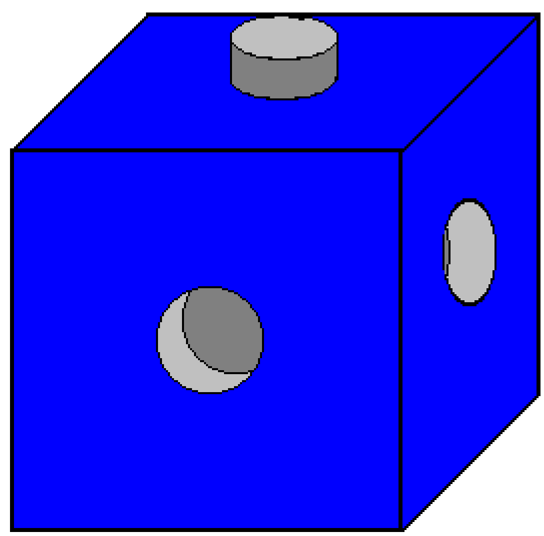
Procedure: Large ASYMMETRIC BLOCKS (*Use 2 cm; uni-connector blocks*)

1. Count out 100 large blocks (*10 stacks of 10 blocks in the box*). Toss them into a working area on the lab floor. Remove the blocks that land with their connector pointing up. Record the number remaining.
2. Toss only the remaining blocks. Remove blocks that land connector up. Record the number remaining.
3. Repeat step #2 eighteen more times, or until no blocks remain.

DATA TABLE: L-BLOCKS

Group: ____

Toss #	***Blocks Remaining***
0	100
1	_____
2	_____
3	_____
4	_____
5	_____
6	_____
7	_____
8	_____
9	_____
10	_____
11	_____
12	_____
13	_____
14	_____
15	_____
16	_____
17	_____
18	_____
19	_____
20	_____



ONE PAGE PER GROUP: This page must be turned in at the end of the lab period so that the instructor can tabulate the data from all groups

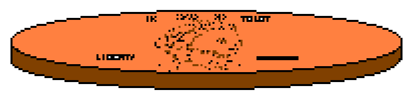
Procedure: COINS

1. Count out 100 pennies. There should be 100 in the small box. Flip them in groups of 5 or so into a large collection area. Remove the pennies that land heads up. Record the number remaining.
2. Flip only the remaining coins in groups of 5. Remove the heads and record the number remaining.
3. Repeat step #2 eighteen more times, or until there are no coins left.

DATA TABLE: COINS

Group: ______________________________

Flip #	***Coins Remaining***
0	100
1	_____
2	_____
3	_____
4	_____
5	_____
6	_____
7	_____
8	_____
9	_____
10	_____
11	_____
12	_____
13	_____
14	_____
15	_____
16	_____
17	_____
18	_____
19	_____
20	_____



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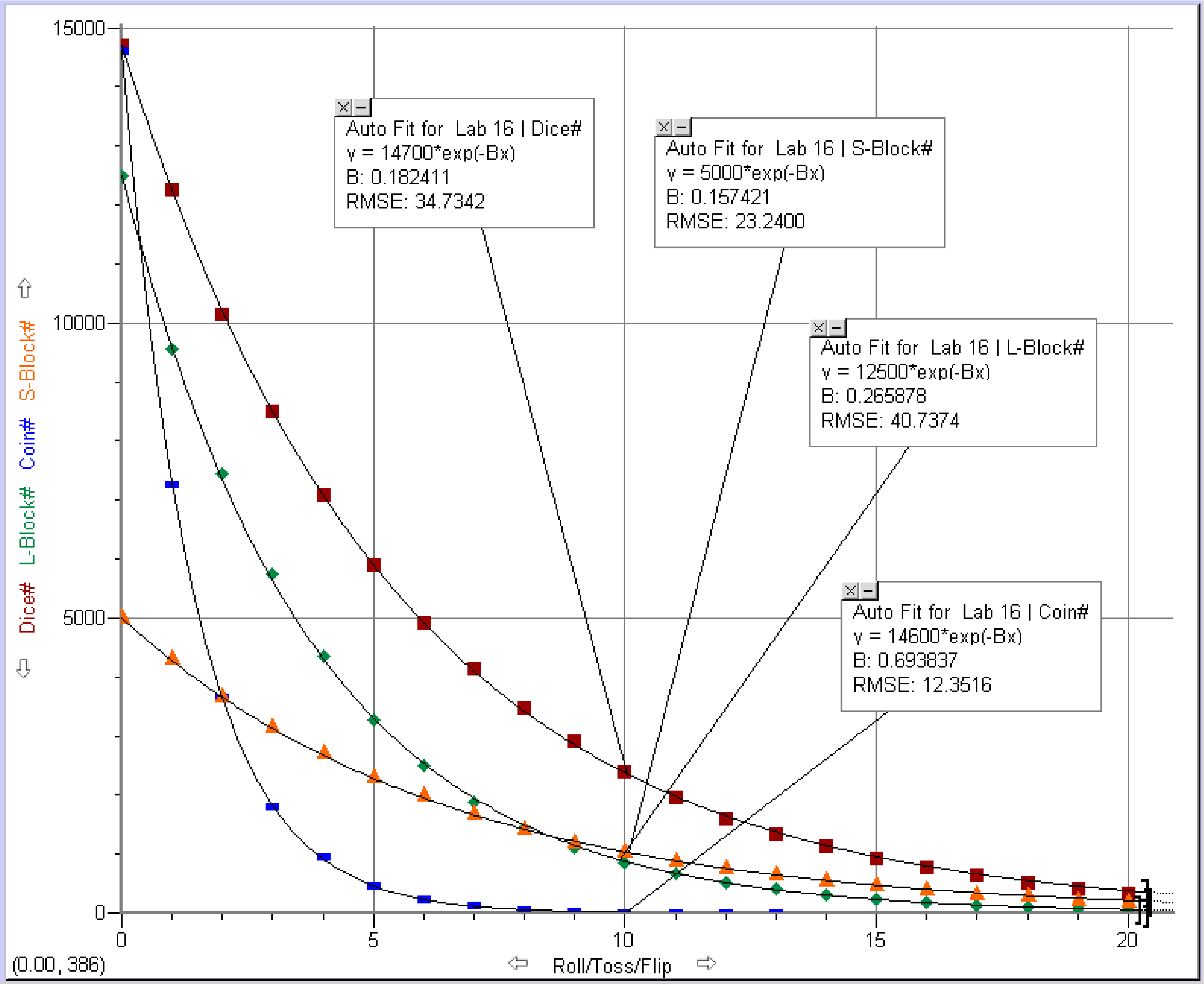
	R/T/F	Dice#	S-B#	L-B#
1	0	14700	5000	1
2	1	12265	4308	
3	2	10156	3645	
4	3	8503	3152	
5	4	7082	2704	
6	5	5905	2281	
7	6	4918	1976	
8	7	4144	1670	
9	8	3474	1412	
10	9	2910	1197	
11	10	2401	1029	
12	11	1961	878	
13	12	1607	759	
14	13	1337	638	
15	14	1129	543	
16	15	924	454	
17	16	777	381	
18	17	633	314	
19	18	508	271	
20				

GRAPH I - Lab 16

Number Remaining vs R/T/F

Exponential Fit
Dice#
S-Block#
L-Block#
Coin#

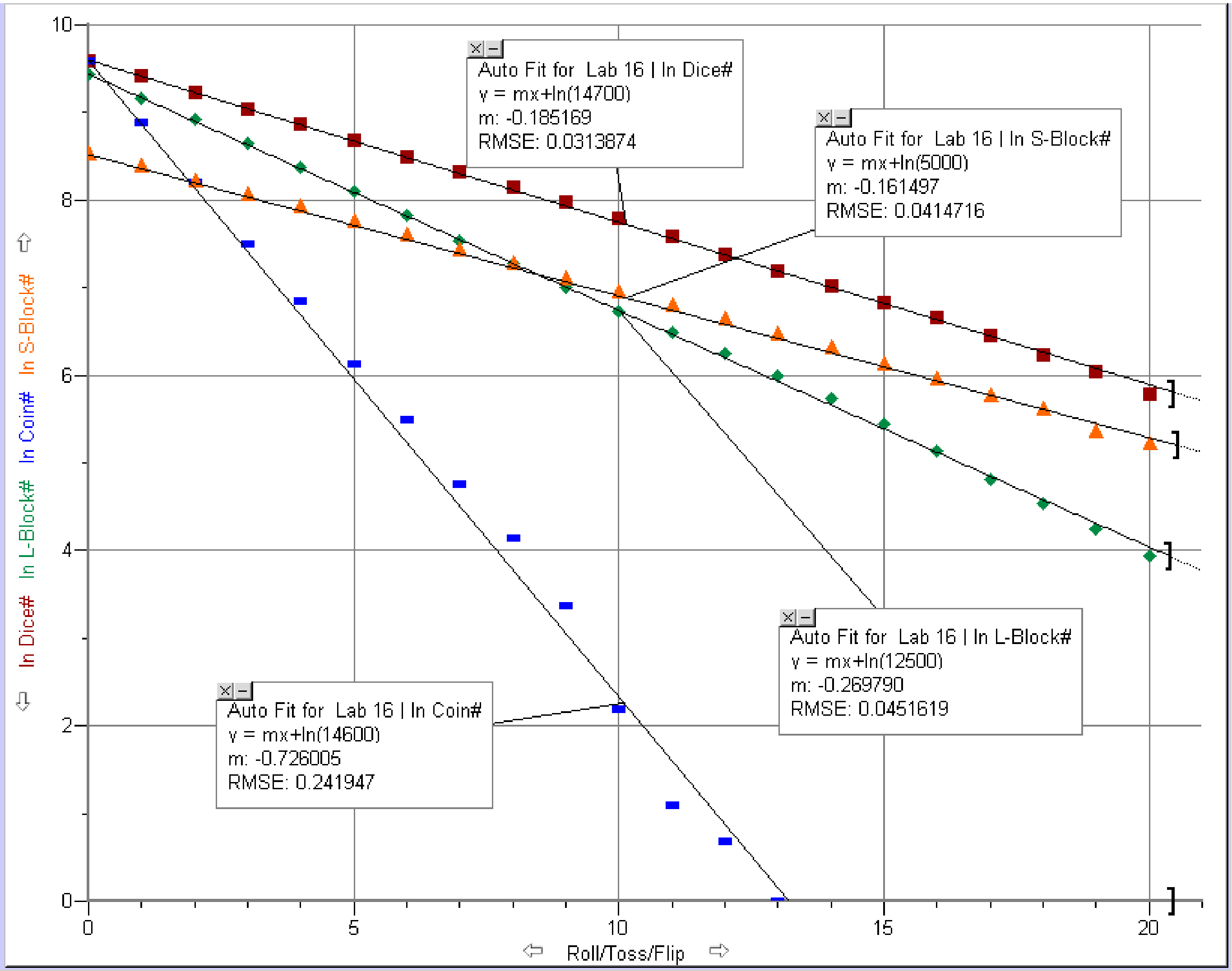


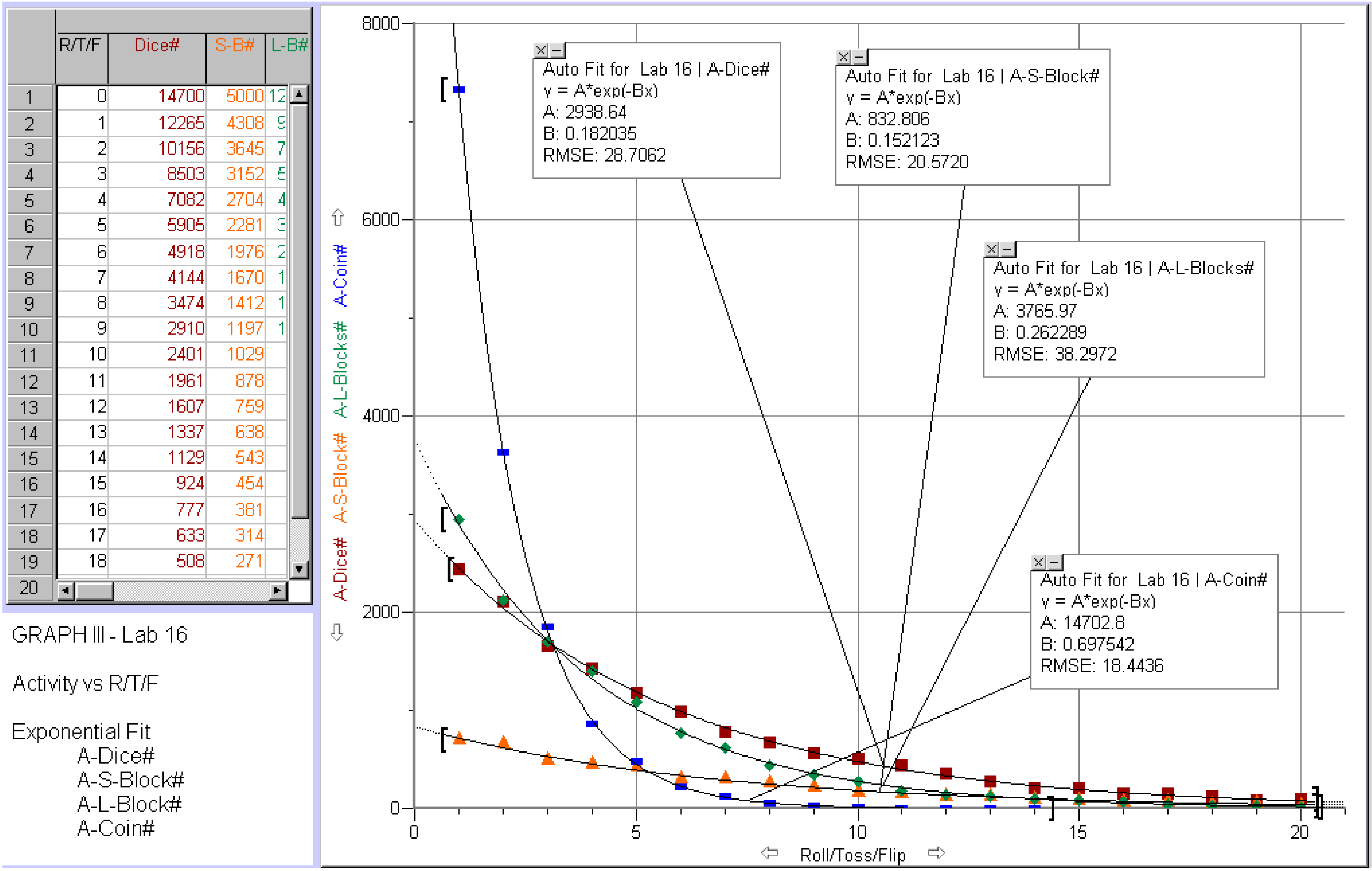
	R/T/F	Dice#	S-B#	L-B#
1	0	14700	5000	12
2	1	12265	4308	9
3	2	10156	3645	7
4	3	8503	3152	5
5	4	7082	2704	4
6	5	5905	2281	3
7	6	4918	1976	2
8	7	4144	1670	1
9	8	3474	1412	1
10	9	2910	1197	1
11	10	2401	1029	[illegible]
12	11	1961	878	[illegible]
13	12	1607	759	[illegible]
14	13	1337	638	[illegible]
15	14	1129	543	[illegible]
16	15	924	454	[illegible]
17	16	777	381	[illegible]
18	17	633	314	[illegible]
19	18	508	271	[illegible]
20				

GRAPH II - Lab 16

LN Number Remaining vs R/T/F

Linear Fit
LN Dice#
LN S-Block#
LN L-Block#
LN Coin#





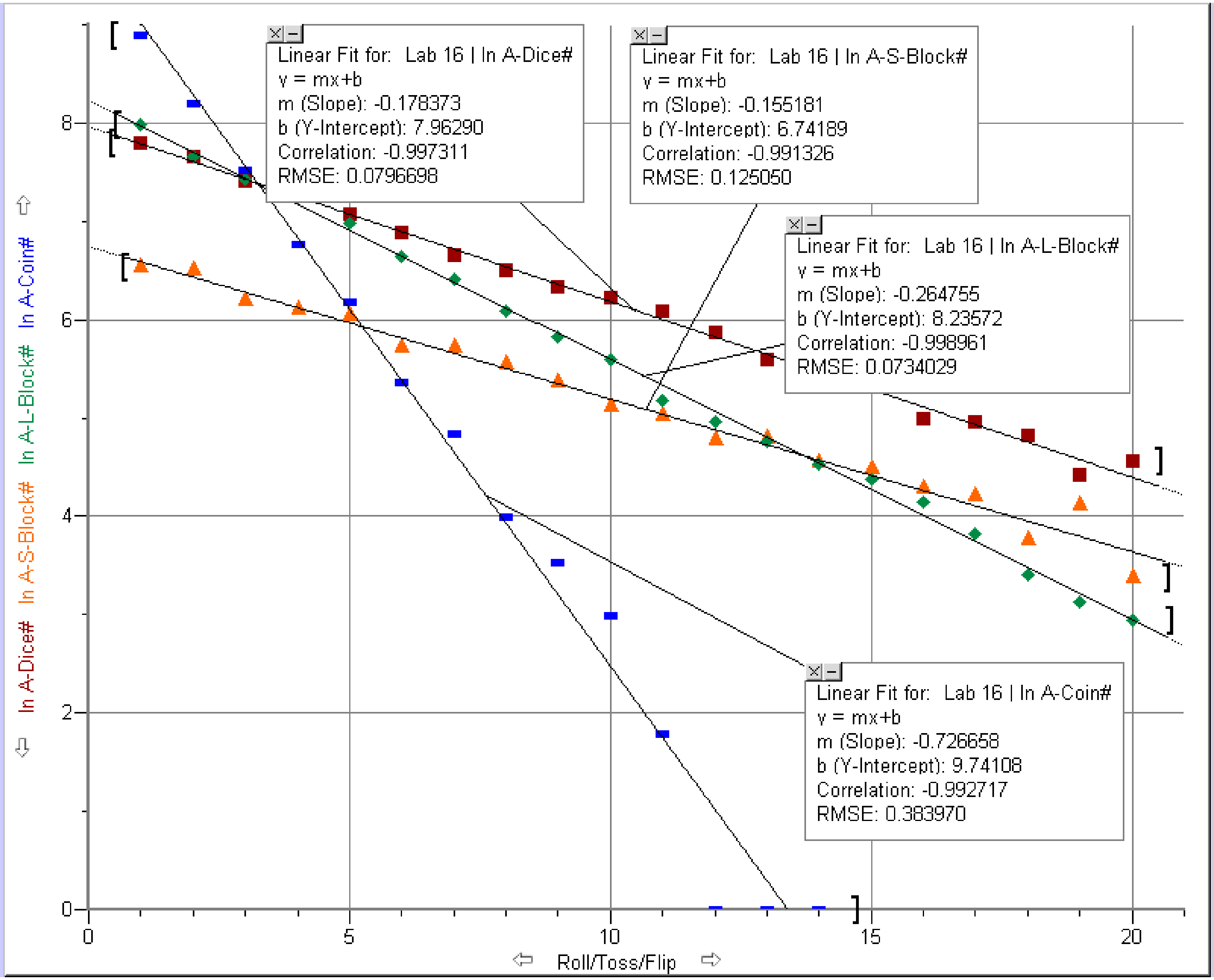
	R/T/F	Dice#	S-B#	L-B#
1	0	14700	5000	12
2	1	12265	4308	9
3	2	10156	3645	7
4	3	8503	3152	5
5	4	7082	2704	4
6	5	5905	2281	3
7	6	4918	1976	2
8	7	4144	1670	1
9	8	3474	1412	1
10	9	2910	1197	1
11	10	2401	1029	
12	11	1961	878	
13	12	1607	759	
14	13	1337	638	
15	14	1129	543	
16	15	924	454	
17	16	777	381	
18	17	633	314	
19	18	508	271	
20				

	R/T/F	Dice#	S-B#	L-B#
1	0	14700	5000	12
2	1	12265	4308	9
3	2	10156	3645	7
4	3	8503	3152	5
5	4	7082	2704	4
6	5	5905	2281	3
7	6	4918	1976	2
8	7	4144	1670	1
9	8	3474	1412	1
10	9	2910	1197	1
11	10	2401	1029	
12	11	1961	878	
13	12	1607	759	
14	13	1337	638	
15	14	1129	543	
16	15	924	454	
17	16	777	381	
18	17	633	314	
19	18	508	271	
20				

GRAPH IV - Lab 16

LN Activity vs R/T/F

Linear Fit
LN A-Dice#
LN A-S-Block#
LN A-L-Block#
LN A-Coin#



Lab 16

	R/T/F	Dice#	S-B#	L-B#	Coin#	ln Dice#	ln S-B#	ln L-B#	ln Coin#	A-Dice#	A-S-B#	A-L-B#	A-C#	ln A-Dice#	ln A-S-B#	ln A-L-B#	ln A-C#
1	0	14700	5000	12500	14600	9.596	8.517	9.433	9.589								
2	1	12265	4308	9556	7276	9.415	8.368	9.165	8.892	2435	692	2944	7324	7.798	6.540	7.988	8.899
3	2	10156	3645	7432	3652	9.226	8.201	8.914	8.203	2109	663	2124	3624	7.654	6.497	7.661	8.195
4	3	8503	3152	5739	1809	9.048	8.056	8.655	7.501	1653	493	1693	1843	7.410	6.201	7.434	7.519
5	4	7082	2704	4343	942	8.865	7.902	8.376	6.848	1421	448	1396	867	7.259	6.105	7.241	6.765
6	5	5905	2281	3265	459	8.684	7.732	8.091	6.129	1177	423	1078	483	7.071	6.047	6.983	6.180
7	6	4918	1976	2495	244	8.501	7.589	7.822	5.497	987	305	770	215	6.895	5.720	6.646	5.371
8	7	4144	1670	1883	117	8.329	7.421	7.541	4.762	774	306	612	127	6.652	5.724	6.417	4.844
9	8	3474	1412	1446	63	8.153	7.253	7.277	4.143	670	258	437	54	6.507	5.553	6.080	3.989
10	9	2910	1197	1109	29	7.976	7.088	7.011	3.367	564	215	337	34	6.335	5.371	5.820	3.526
11	10	2401	1029	841	9	7.784	6.936	6.735	2.197	509	168	268	20	6.232	5.124	5.591	2.996
12	11	1961	878	663	3	7.581	6.778	6.497	1.099	440	151	178	6	6.087	5.017	5.182	1.792
13	12	1607	759	521	2	7.382	6.632	6.256	0.693	354	119	142	1	5.869	4.779	4.956	0.000
14	13	1337	638	404	1	7.198	6.458	6.001	0.000	270	121	117	1	5.598	4.796	4.762	0.000
15	14	1129	543	311		7.029	6.297	5.740		208	95	93	1	5.338	4.554	4.533	0.000
16	15	924	454	232		6.829	6.118	5.447		205	89	79		5.323	4.489	4.369	
17	16	777	381	169		6.655	5.943	5.130		147	73	63		4.990	4.290	4.143	
18	17	633	314	123		6.450	5.749	4.812		144	67	46		4.970	4.205	3.829	
19	18	508	271	93		6.230	5.602	4.533		125	43	30		4.828	3.761	3.401	
20	19	425	210	70		6.052	5.347	4.248		83	61	23		4.419	4.111	3.135	
21	20	329	181	51		5.796	5.198	3.932		96	29	19		4.564	3.367	2.944	

GRAPH IV - Lab 16

Data Table

ALL 17 Columns