

Name: _____ **Period:** _____ **Due Date:** _____
Lab Partners: _____



Purpose: To investigate the factors affecting the Simple Harmonic Motion of a hanging mass on a spring. Explore static and dynamic methods of measuring the spring constant of a spring or a combination of springs.

Part One:

Procedure: Determination of k for your springs using static forces (hanging masses).

1. Hang a weight on your spring #1 and carefully and accurately measure the total length of the spring.
2. Calculate the mass and weight of the washers. Record the mass to the nearest 1 gram = 0.001 kg and the length to the nearest 0.1mm = 0.01 cm = 0.0001 m).
3. Repeat steps 1- 2 until you've tested six different masses on the spring.
4. Repeat steps 1- 3 for your second spring employing the exact same weights used on the first spring.
5. Connect the two springs end-to-end to create a double spring. Hang the combined spring with the lightest one on the bottom.
6. Repeat steps 1- 3 for the doubled spring employing the same weights used on the first spring.

Data Table I

Part One Data: Mass of 100 washers = _____ kg; Mass of one washer = _____ kg

Number of washers	Mass (kg)	Weight (newtons)	Spring #1 length (meters)	Spring #2 length (meters)	Spring #1+#2 length (meters)
3	_____	_____	_____	_____	_____
5	_____	_____	_____	_____	_____
7	_____	_____	_____	_____	_____
8	_____	_____	_____	_____	_____
9	_____	_____	_____	_____	_____
10	_____	_____	_____	_____	_____
			smaller spring	larger spring	

Analysis of Part One:

Graph A: Use *Graphical Analysis* and put all three sets of spring-length data on the same graph. A graph of the

Length of the Spring (vertical axis) vs. Force (equals mg , horizontal axis)

will be a straight line. The equation of this line is $L = L_0 + \Delta L = L_0 + mg/k$, therefore the slope of this line will be $1/k$, where k is the spring constant. We are mostly interested in the k values. We will not be making any further use of the L_0 values in this lab.

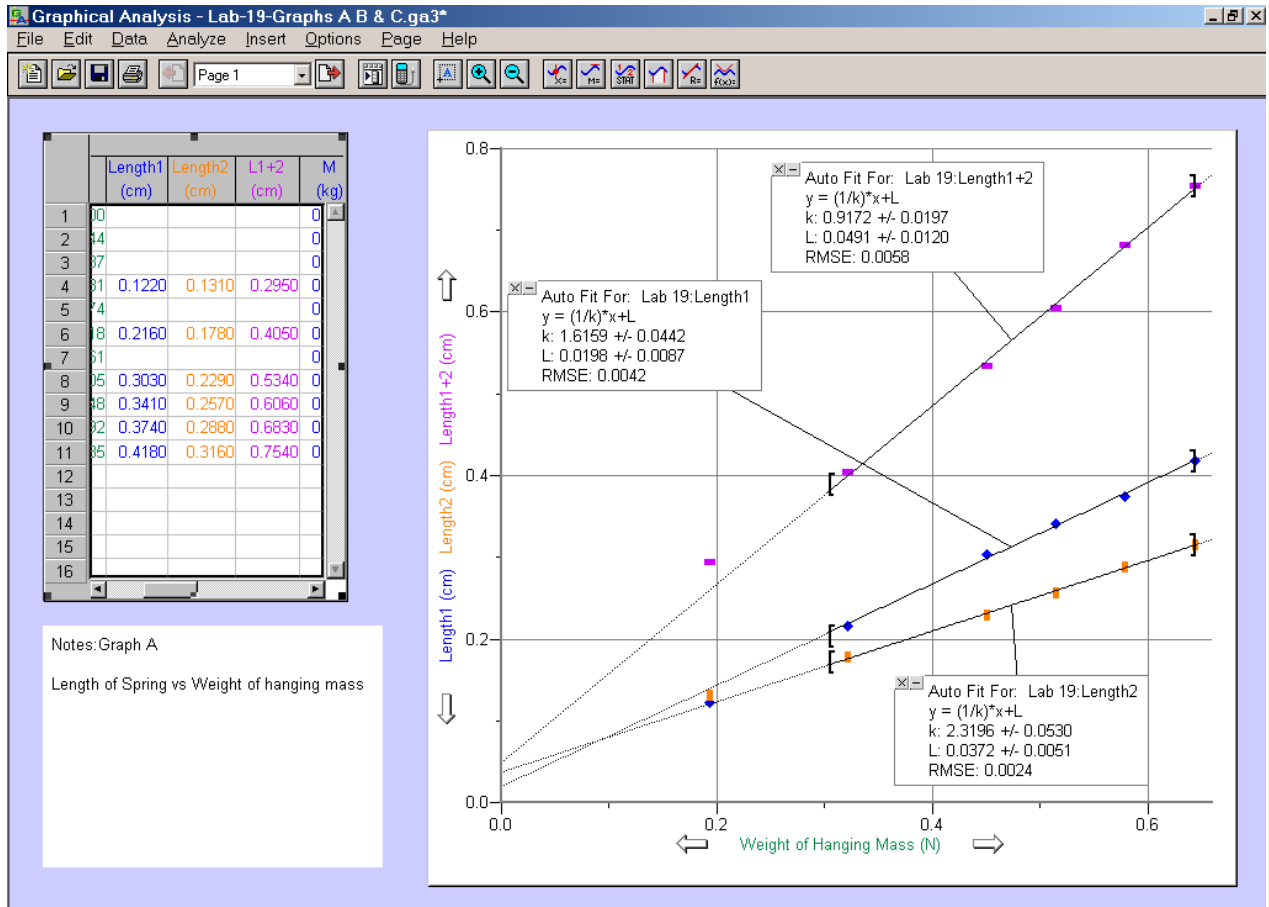
Select Auto Scale from 0 for both the x-axis and the y-axis, and Make sure that you can see the y-intercepts of all three lines, and Make sure you can see all the data points. (*In the graph below the interval ['s are incorrectly set!)* Select Curve Fit from the Analyze Menu. Redefine the straight-line function to read as $(1/k)*x + L$. Fit each line individually and then display all three on the final graph.

Title the graph: **Lab 19 - Graph A**

Print the whole screen so that the graph and the data on it are visible in the printout.

Manually label all three lines on the printout so it's clear that you know which line is spring #1, which is spring #2, and which is the double spring.

Here is a sample graph which does not include the hand labeling.



Part Two:

Procedure: Dynamic determination of k using Simple Harmonic Motion (SHM).

1. Place a mass on your spring #1 and determine the period of the simple harmonic motion that results.
 An easy and satisfactory method is to find the time for 20 periods and then divide by 20 to get the time for one period, **T**. This gives you accuracy to 0.05 sec using an ordinary wristwatch with a second hand. Record the mass and period in the data table. Do not over-stretch the springs; either with too much weight or by pulling down too far. A tiny (~1/2 inch) displacement is sufficient.
2. Repeat the determination of the period for seven additional masses and record the data in the data table.
3. Connect the two springs end-to-end and repeat the SHM experiment using the same eight masses. Hang the smaller, lighter spring below the heavier spring. Record your measurements in Data Table II.

Data Table II

Part Two Data:

Hanging weights		Single Spring (#1 only)		Double Spring (#1 and #2)	
Number of washers	Mass (kg)	Period (sec)	Period ² (sec ²)	Period (sec)	Period ² (sec ²)
___ 3 ___	_____	_____	_____	_____	_____
___ 4 ___	_____	_____	_____	_____	_____
___ 5 ___	_____	_____	_____	_____	_____
___ 6 ___	_____	_____	_____	_____	_____
___ 7 ___	_____	_____	_____	_____	_____
___ 8 ___	_____	_____	_____	_____	_____
___ 9 ___	_____	_____	_____	_____	_____
___ 10 ___	_____	_____	_____	_____	_____

Analysis of Part Two:

Graph B: Use *Graphical Analysis* to put both the single-spring and double-spring data on the same graph. A graph of the

The Period, T (vertical axis) vs the mass, m (horizontal axis)

will be a parabola on its side through the origin. The equation of this curve is known as the power function. $Y = A \cdot x^B$. We expect that $B = \frac{1}{2}$. The result should be parts of two parabolas on their sides and opening to the right.

Set both axes to Auto Scale from zero. (In the graph below the interval ['s are incorrectly set!)

Find the *power function* in the Curve Fit menu.

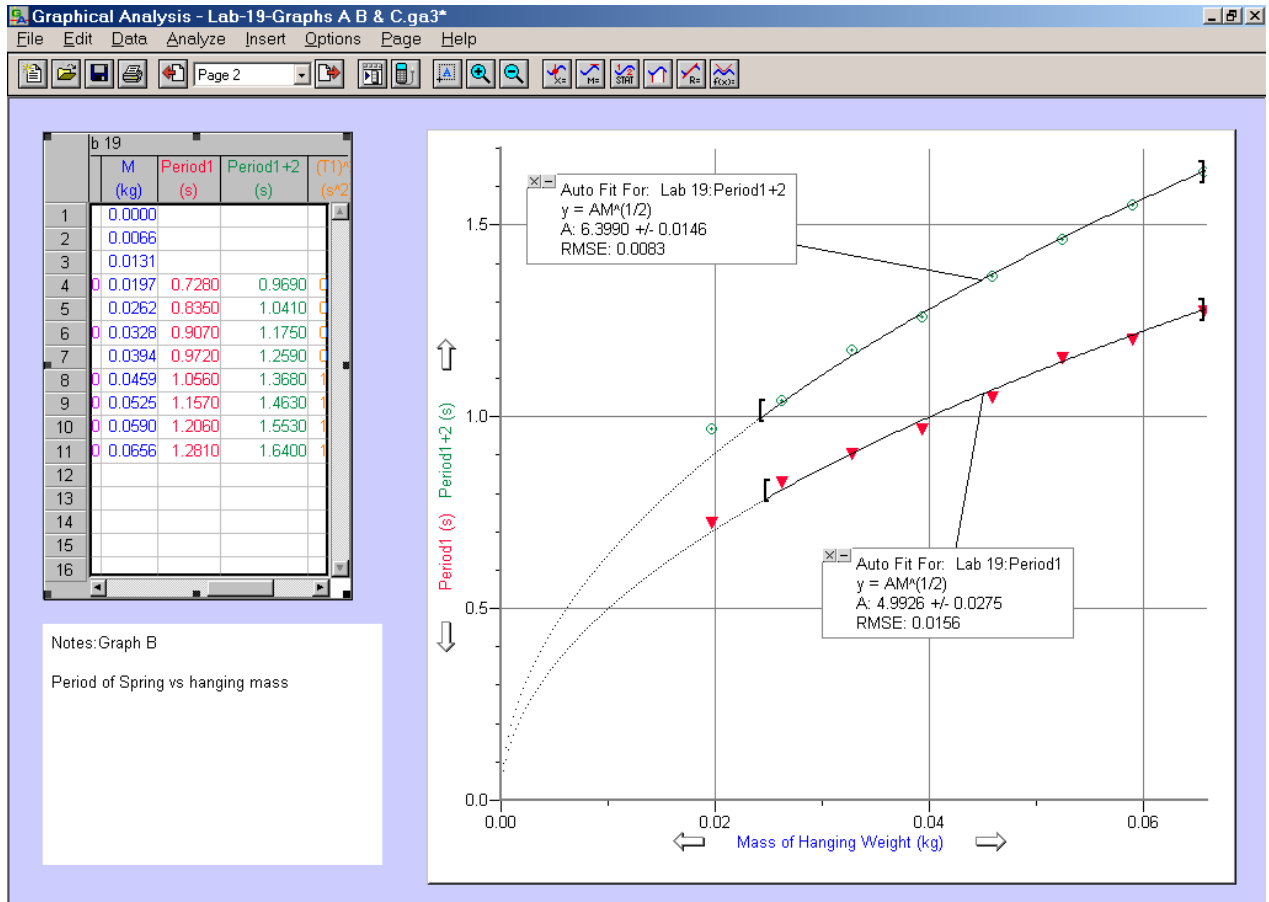
Modify the Power Function to read $A \cdot x^{(1/2)}$ (square root function).

Fit both data sets individually and then display them on the final graph.

Title it: **Lab 19 - Graph B**

Print the whole screen so that the graph and the data on it are visible in the printout.

Manually label both lines on the printout so it's clear which line is spring #1, and which is the double spring.



Analysis of Part Two: continued

Graph C: Create new calculated columns for period squared and have *Graphical Analysis* calculate the values for you. Then use *Graphical Analysis* to make a third graph of

The Period squared, T^2 (vertical axis) vs the mass, m (horizontal axis).

The result should be two straight lines through the origin.

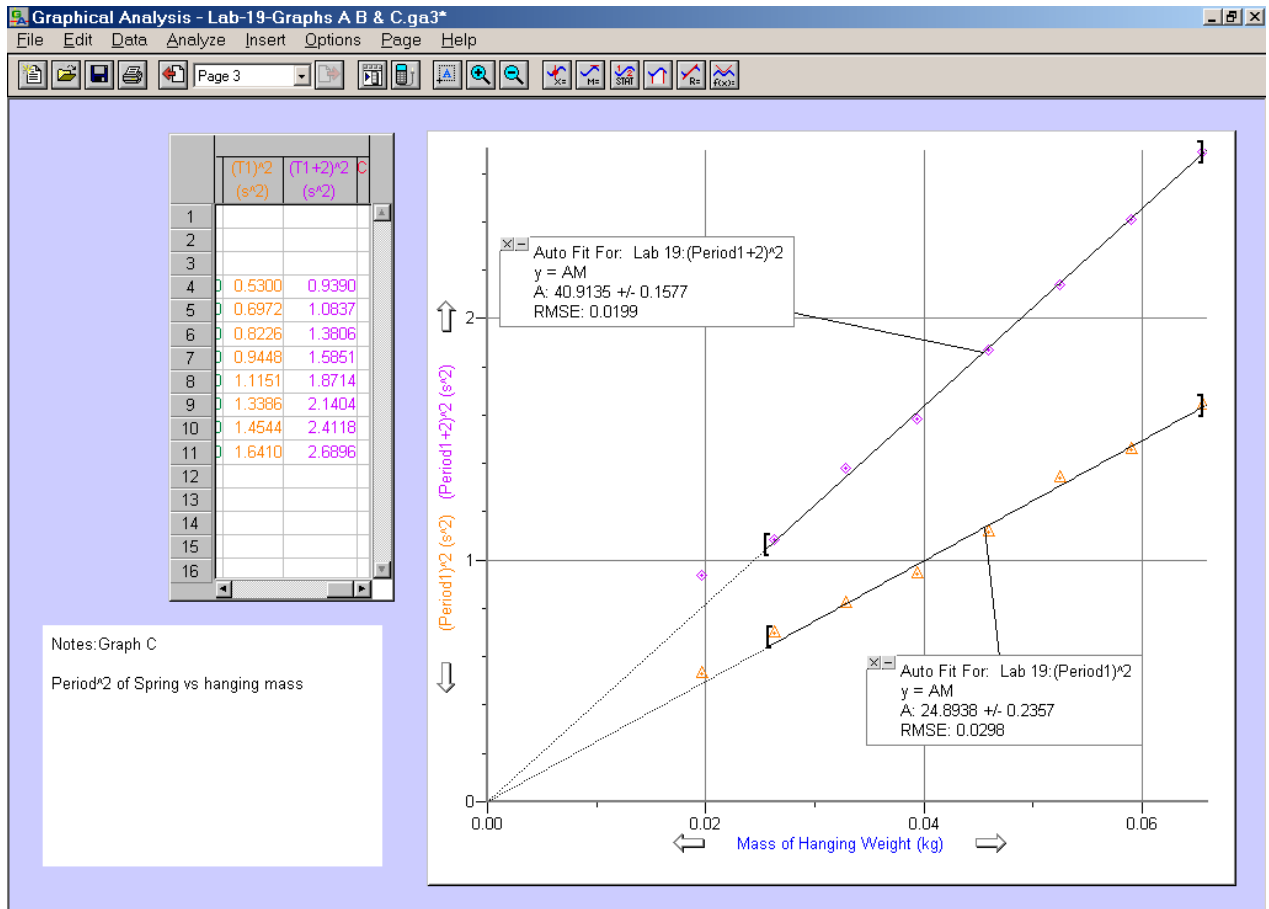
Set both axes to Auto Scale from zero. (*In the graph below the interval ['s are incorrectly set!*)

For the function, select the proportional function under the Curve Fit menu; $A \cdot x$ (a straight line through the origin).

Title it: **Lab 19 - Graph C**

Print the whole screen so that the graph and the data on it are visible in the printout.

Manually Label the lines on the printout so it is clear which is for spring #1 and which is the double spring.



Results and Interpretation:

Graph A:

Spring Constant of spring #1 in **Graph A**: $k_1 =$ _____

Spring Constant of spring #2 in **Graph A**: $k_2 =$ _____

Spring Constant of the double spring in **Graph A**: $k_{\text{Double}} =$ _____

Check Results: $1/k_1 + 1/k_2 =$ _____; $1/k_{\text{Double}} =$ _____; %Diff = _____

The units of k will be horizontal-axis-units / vertical-axis-units = _____ / _____

Graphs B and C:

The following equations describe the simple harmonic motion of a weight on a spring.

$$T = 2\pi\sqrt{\frac{m}{k}} = \sqrt{\frac{4\pi^2}{k}} \cdot \sqrt{m} \quad \text{and} \quad T^2 = 4\pi^2 \frac{m}{k} = \left[\frac{4\pi^2}{k} \right] \cdot m$$

The coefficient, **A**, of the single-spring curve on **Graph B** = _____ = $(4\pi^2/k_1)^{1/2}$.

The coefficient, **A**, of the double-spring curve on **Graph B** = _____ = $(4\pi^2/k_{\text{Double}})^{1/2}$.

The slope, **A**, of the single-spring line on **Graph C** = _____ = $(4\pi^2/k_1)$.

The slope, **A**, of the double-spring line on **Graph C** = _____ = $(4\pi^2/k_{\text{Double}})$.

The coefficient of the **Graph B** parabola equals $\sqrt{(4\pi^2/k)}$, while the slope of the line in **Graph C** = $4\pi^2/k$. Each of these best-fit parameters can be used to find the spring constant, k. Fill-in your results here.

From **Graph B**, $k_1 =$ _____ and, $k_{\text{Double}} =$ _____.

From **Graph C**, $k_1 =$ _____ and, $k_{\text{Double}} =$ _____.

The results from graphs **B** and **C** should be very close. If they are not very close then double check your calculations (*double check your measurements, if necessary*) and make sure you have not made an error. We could use either one of these sets of k values or even the average of the two. For consistency you will use only the values of k_1 and k_{Double} from **Graph C** in the analysis that follows.

The units of k from **Graph C** will be horizontal-axis-units / vertical-axis-units = _____ / _____

Report the %Difference between k_1 determined from **Graph A** and k_1 determined from **Graph C**.

%Difference = $100\% \times |k_{1(A)} - k_{1(C)}| / k_{1(A)} =$ _____

The value of k_2 , the second spring's spring constant, can now be determined from these results. It can be calculated from the measured values of k_1 and k_{Double} . The next section will show you how. Then compare that result with the k_2 determined from **Graph A**. You've already seen how well the **SHM**-method works for finding k_1 . Now you will find out how well the **SHM**-method works for finding the spring constant of an "unknown" spring in a pair of springs connected in end to end.

Derivation of the Expression for the "Other Spring's" Spring Constant.

Two springs are connected end-to-end and support a hanging weight. Always hang the lighter spring below the heavier spring (*assuming there is a notable difference between the masses of the two springs*) when attempting this type of a measurement for this type of analysis.

Springs in series each support the entire weight of the hanging mass. Therefore, the total stretch is the sum of the stretch of each spring. This can be shown in a short derivation. It starts with a statement that the total length of the stretched springs equals the sum of the lengths of the individual stretched springs:

$$\begin{aligned} L_{\text{Double}} &= L_1 + L_2 \\ L_{0,\text{Double}} + \Delta L_{\text{Double}} &= (L_{0,1} + \Delta L_1) + (L_{0,2} + \Delta L_2) \\ (L_{0,1} + L_{0,2}) + \Delta L_{\text{Double}} &= (L_{0,1} + L_{0,2}) + (\Delta L_1 + \Delta L_2) \\ \Delta L_{\text{Double}} &= \Delta L_1 + \Delta L_2 \end{aligned}$$

Since, according to Hooke's Law, " ΔL " can be replaced with F/k this becomes

$$F_{\text{Double}} / k_{\text{Double}} = F_1 / k_1 + F_2 / k_2$$

The forces are all same and equal to the mass of the hanging weight ($F_{\text{Double}} = F_1 = F_2 = mg$: *Assuming we can ignore the weight of the lower spring hanging from the upper spring*), therefore, both sides of the equation can be divided by that common force to yield an equation relating the spring constants of the individual springs to the apparent spring constant of the combined spring; (*This equation can be generalized for three or more springs.*)

$$1/k_{\text{Double}} = 1/k_1 + 1/k_2$$

(This should look familiar by now since we've seen the same form of equation a number of times already. It is exactly the same form we encountered for finding the focal length of a mirror or lens from the image and object distances, for finding the equivalent capacitance for two capacitors in series, and for finding the equivalent resistance for two resistors in parallel.)

Remember the special solution to this type of equation when only two resistors, capacitors, mirrors or springs are present:

$$\text{If} \quad 1/s = 1/a + 1/b$$

$$\text{Then} \quad s = ab / (a + b) \quad \text{or} \quad a = sb / (b - s)$$

The mnemonic for this is "the product over the sum" or T-POTS.

Graph C gives you k_1 and k_{Double} , therefore, $k_2 =$ _____.

Now you can report a percentage difference between the value of k_2 determined from **Graph A** and the value of k_2 determined indirectly from **Graph C**.

$$\% \text{Difference in } k_2 = 100\% \times |k_{2(A)} - k_{2(C)}| / k_{2(A)} = \underline{\hspace{2cm}}$$

[NOTE: If you find that you accidentally confused the two springs do not worry. If that happened and you don't discover it until you are in the middle of the Analysis, all is not lost. You should however, go back to Data Table I and re-label the two columns labeled "Spring #1" and "Spring #2". Also, re-label the tops of the Data Columns in Graphical Analysis and the lines on your graphs. The two springs you were given should have different spring constants, so it should be easy to tell which is which from a comparison of the spring constants, if not from appearance alone. To interpret the data properly it is essential that your data reflect accurately what you've done, and that all the data be labeled correctly. For best results the smaller, lighter spring should be spring #1. Spring #1 will also have the smaller spring constant.]

Sample Calculations: Show the calculation for the data from the earliest example of each calculation used in filling your data tables or analyzing the results. *Show the expression and the numbers as well as the final numerical result.* Don't forget to **show the units** of every number in the calculation and the units of the final answer. **Put a box around the answer.**

Convert mass in grams to mass in kilograms and then to weight in newtons.

Convert twenty periods to one period and then to period squared.

Find k from the slope of the line in *Graph A*.

Find k from the coefficient of \sqrt{m} in *Graph B's* equation for a parabola [$T = \sqrt{4\pi^2 / k} \bullet \sqrt{m}$].

Find k from the slope of the line in *Graph C* [$T^2 = (4\pi^2 / k) \bullet m$].

Calculating a %Difference.

Find k_2 from k_{Double} and k_1 .

Any others? Use additional paper if necessary.

Questions for Analysis: (In addition to the questions on these two pages (Questions 5-8), there are also four more questions on WebAssign (Questions 1- 4).

5. Unit analysis. The slope of the line in **Graph A** has units of meters/newton. The slope of the line in **Graph C** has different units. Show that these two sets of units are equal to each other. (1/2 point)

6. Indicate if each of the following describes the graph of a straight line.

[Specify **yes** if linear or **no** if not linear for each case] (1/8th point each)

a) T^2 vs. k _____

b) T^2 vs. m _____

c) T vs. k _____

d) m vs. k _____

e) T^2 vs. \sqrt{m} _____

f) T vs. $1/k$ _____

7. Look at the period of your double spring system and compare it to the individual springs.

Did the **period of the double spring** increase or decrease compared to the separate springs? _____
(1/8th point)

What does this indicate about **k of the double spring**? Did k increase or decrease? _____
(1/8th point)

Linking two identical springs end-to-end will result in a combined spring constant (k_{Double}) that equals:

Circle the correct answer (1/4th point)

a) k_{original} divided by 2

b) k_{original} divided by $\sqrt{2}$

c) k_{original} unchanged

d) k_{original} multiplied by $\sqrt{2}$

e) k_{original} multiplied by 2

8. Suppose you are driving in a car on a street with speed bumps every 3.0 meters. Your car has a mass of 1300 kg and sits on springs with $k = 84,000$ N/m. The car has only the springs, and no shock absorbers. As you hit the first bump the springs are given an initial compression and then starts to expand. Each time you hit another speed bump the springs receive an additional displacement that can either compress or expand the springs depending on where they are in their cycle. The bouncing will be reinforced if the frequency with which you hit the speed bumps equals the frequency of your car springs. This matching of frequencies in two vibrators is called resonance. Resonance might be called "constructive reinforcement" or "constructive amplification."

(1/8th point each)

A-What is the driving speed that will produce optimum resonance? (*Resonance is strongest when the frequencies of the spring's bounce and the car hitting the speed bumps are the identical. If the car goes faster some of the speed bumps will partially cancel the amplitude of the bounce in the springs and decrease the maximum size of the bounce. If the car goes slower, the amplitude of the spring bounce will have time to decrease because of friction and that will also decrease the maximum size of the bounce.*)

B-What would happen if the car maintained this speed but the bumps were twice as far apart?

C-What would happen if the car maintained this speed but the bumps were half as far apart?

D-Many other speeds will also produce resonance, but it will be less efficient. Explain how these speeds are related to each other and to the speed you calculated in part **A**.

E-Why should resonance be avoided in cases involving physical systems like this one with a car and speed bumps, or a bridge with soldiers marching in step across it, or buildings swaying in synchronization to the vibrations from an earthquake, etc?

THESE QUESTIONS HAVE BEEN REWRITTEN AND ARE NOW ON WEBASSIGN

Questions for Analysis: *(Except for fill-in the blank and multiple-choice, show your work on a separate sheet of paper. No points are awarded unless the work is shown clearly and completely. Attach the extra page immediately following this page in your final report.)*

WA1 Given a spring with $k = 22.1 \text{ N/m}$, calculate

- the period of a 3.2 kg mass bouncing in simple harmonic motion at the end of this spring?
- the frequency of the simple harmonic motion of this spring and mass combination?

WA2 When an object simply hangs at rest on a spring, $F = k \Delta L = mg$, or $\Delta L / g = m/k$. If an unknown mass stretches an unknown spring by 0.123 m , what is the period when the mass oscillates in SHM on the end of the spring?

WA3 When the mass on a spring is doubled, the period of simple harmonic motion will be multiplied by

- 2
- $\sqrt{2}$
- 1
- $1/\sqrt{2}$
- $1/2$

WA4 A given mass on a given spring has a period of 2.0 sec in SHM. When you add 2.0 kg more to the mass, the period changes to 3.0 sec . What was the original mass? What is the spring constant of the spring?