

Name: _____ Period: _____ Due Date: _____

TWO-DIMENSIONAL COLLISIONS

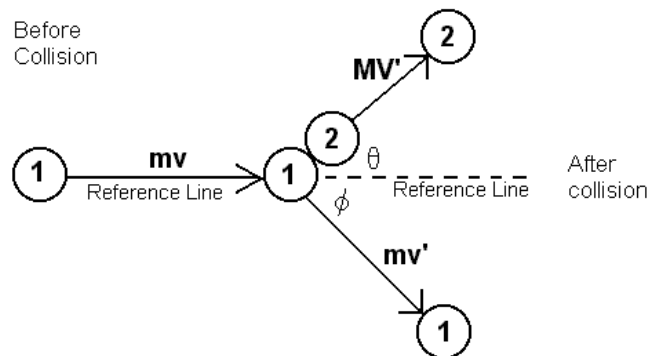
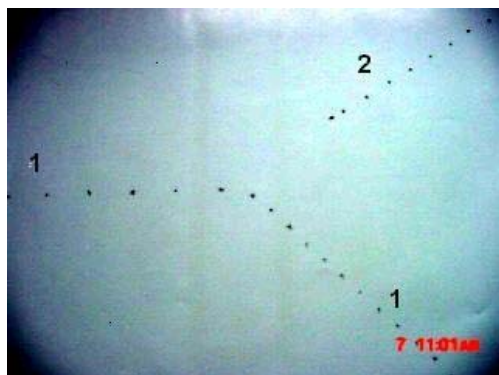
Purpose: To examine momentum conservation in a two-dimensional collision using vector addition.

Procedure:

1. Push two air pucks into a collision on a spark table. One mass (m) is moving before the collision, while the other (M) is stationary. After the collision both pucks move away from the collision point. Record the information here.

mass₁ = m = _____ kg; mass₂ = M = _____ kg; spark frequency = _____ Hz (or sec⁻¹)

2. Orient the side of the paper with the spark marks so that the marks appear as shown below left.



Above left is a picture of the spark traces of one such collision. The diagram above right illustrates the positions of the two pucks before the collision, at the instant of collision and after the collision. It also shows the momentum vector of each puck before and after the collision. In the section that follows you will make measurements on the spark diagram from which you will determine the magnitude and direction of these three momentum vectors.

Note a couple of conventions we've adopted for this analysis: The small letters, m and v , apply to the puck#1, which was moving before the collision. The large letters, M and V , apply to puck#2, which was initially at rest. The velocities are un-primed, v and V , before the collision and primed, v' and V' , after the collision. Note, before the collision, $V = 0.00$ m/s, because puck#2 was not moving until the instant after the collision. The spark frequency, the gaps between the dots, and the direction angles define the momentum vectors.

Note: The sheet of paper with the spark marks is your **primary data** or **source data**. It is essential that all the relevant measurements and calculations be written on that paper. If all else is lost, you could reconstruct your entire analysis from the source data and your original measurements recorded there. Do not rely on your memory to store the information that is rightly part of the primary data record. Then, you need to copy the average gap lengths and rebound angles into this report. **Do not lose the primary data sheet. You must turn it in with the write-up.** All written information on the data sheet must be neat and legible.

3. **Measure the gap lengths between spark points.** Identify the collision points and measure at least five full gap lengths out from the collision point along all three trajectories. You need to find the velocity of puck#1 an instant before the collision and the velocities of both pucks in the instant after the collision. Puck#1 is likely to be slowing just before the collision and both pucks are likely to start slowing down immediately after the collision.

Use as many gaps as you can, provided the gap distances are nearly constant. The farther from the collision point you get the more likely the gaps are to differ from those nearer the collision point. The speed is always changing due to friction between the puck and the paper. What you need to determine are the velocities just before and just after the collision. Therefore, the gap distances that best represent the correct velocities are the gaps closest to the collision point. Gaps that contain the collision point cannot be used.

You must use the gap closest to the collision point. And, you may not skip any gaps. If the gap lengths change too quickly, you will not be able to use all five measured gaps. You might be able to use only the two or three gaps closest to the collision point. You might, in the worst case, be able to use only the one gap closest to the collision point. Deciding how many gaps to use is a matter of judgment. Your instructor is available to discuss your situation, if you would like a second opinion, before making your final determination.

Once you've decided which gaps to use, make one measurement of the entire distance across those gaps you decided to include. This is better than adding and averaging the individual gap measurements. There is less error in one long measurement than in several short measurements. Divide the measured length by the number of gaps to determine the average gap length along each trajectory.

Show the measurements, the calculations, and the results on the spark paper. Then copy the results of the average gap length into the spaces provided below.

Gap_{#1} (before collision) = _____ meters Gap_{#1}' (after collision) = _____ meters

Gap_{#2} (before collision) = 0.0000 meters Gap_{#2}' (after collision) = _____ meters

4. **The velocity of the puck can be found** by dividing the average gap length by the time between sparks, or, since division by the period is the same as multiplication by the frequency, we can multiply the average gap length by the frequency. Either way you get the same result. Velocities in the range from 0.7 m/s down to 0.1 m/s are typical.

Now calculate the velocity of puck #1 before, and after the collision and the velocity of puck #2 after the collision, using the following equation. Show the calculations and record the results on the spark paper and then copy the results into the spaces provided below.

$$\text{velocity} = (\text{average gap length}) / (\text{time between sparks}) = (\text{average gap length}) \times (\text{spark frequency})$$

(m/s) = (meters / seconds) = (meters x seconds⁻¹)

V (before) = _____ m/sec

V' (after) = _____ m/sec

V (before) = 0.00 m/sec

V' (after) = _____ m/sec

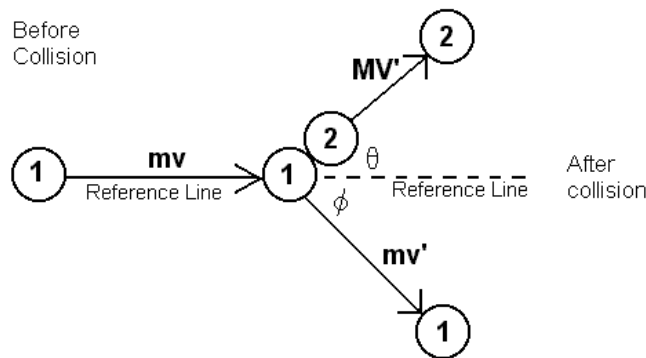
- 4a. **An alternate method of determining velocities**, for students who have the interest and some extra time to devote to this project, is to plot the gap length vs the distance from the collision point. This line can be extrapolated back to the collision point to estimate the instantaneous gap length and thus the instantaneous velocity at the moment of collision. This should be done three times; once along each of the three trajectories. Students interested in trying this particular type of analysis must talk to their instructor before they begin.
5. Now that the velocities before and after the collision are known, multiply the mass times velocity to determine the momentum before and after the collision. These are only the measured magnitudes of the momentum vectors. We'll find the direction angles for these vectors in the next section.

\mathbf{mv} (before) = _____ kg·m/sec \mathbf{mv}' (after) = _____ kg·m/sec

\mathbf{MV} (before) = 0.00 kg·m/sec \mathbf{MV}' (after) = _____ kg·m/sec

6. **Measure the rebound angles** of the two pucks after the collision using the inbound direction as your reference direction (think of it as indicating the direction of the positive x-axis) according to the following diagram.

Mark and label the two angles on the data sheet. Write the measured angles as well as the symbols. Measure the angles to the nearest 1/10th of a degree by estimating between the marks on the protractor. Then copy the values into the spaces below.



In-bound direction of puck #1 = Positive-x
direction = Reference direction = **0.0°**

Rebound angle of puck #2 = θ = _____ ° (θ is a positive angle between 0° and 90°)

Rebound angle of puck #1 = $|\phi|$ = _____ ° (ϕ is a negative angle between 0° and -90°, when measured from a reference line and when used for calculating the components of a vector. When used to measure angles in a triangle, like the vector triangle, it is just like any other angle of a triangle, which is to say, a positive angle. In situations where we need the positive value, it will be indicated as $|\phi|$. To be clear, when we need the negative value, we will explicitly use $-|\phi|$.)

We could have used any direction as our reference direction for measuring these angles. We chose to use the incoming trajectory as our reference direction because that turns out to be a very convenient reference direction. It is very convenient to have one of the vectors pointing along the **0.0°** (the +x-axis) direction because that simplifies other calculations. The vector arithmetic would work just as well if we chose to use any other arbitrary reference direction as the basis for our calculations with vector components.

7. We studied **momentum conservation** in one-dimensional collisions earlier in the year. At that time we used the air tracks and air carts to verify momentum conservation. We used the photogates to monitor the velocities of the carts before and after the collision. This time we are studying collisions in two dimensions. Momentum is still conserved. In this lab we use air-pucks instead of air carts. We use regularly timed electric sparks instead of photogates to monitor the velocities. We perform our vector addition on the momentum vectors using x and y components instead of simple addition. The most important difference between studying one-dimensional and two-dimensional collisions is that we must use two-dimensional vectors to describe the momentum of each cart before and after the collision.

You will verify momentum conservation in your two-dimensional collision four ways;

- A) Vector addition of the momentum vectors using the vector component method,
- B) Verify the Law of Sines by checking the ratios in the triangle of vector momenta,
- C) Verify the Law of Sines by comparing the measured and calculated angles in the triangle of vector momenta,
- D) Verify the Law of Cosines by comparing the measured and calculated sides in the triangle of vector momenta.

In the component method of vector addition, measurements of the angles and distances are made with respect to a coordinate system. That means you must remember to assign appropriate positive (+) and negative (-) signs to the measured angles. The magnitude of a vector is always positive. The components of a vector can be positive or negative.

Specifically, $|\phi|$ needs a negative sign, $-|\phi|$, when used in vector-component calculations, since it represents a clockwise angle relative to the reference line. Furthermore, while both components of the velocity vector for puck #2 will be positive, the y-component of the velocity vector of puck #1 after the collision will be negative.

If momentum is conserved, then the sum of the momentum vectors before the collision will equal the sum of the momentum vectors after the collision. The same is true for the vertical and horizontal components.

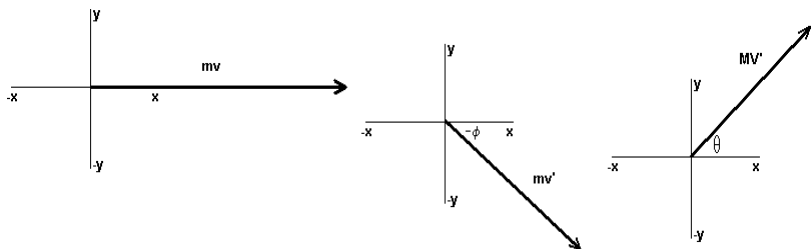
Avoid excessive rounding; keep *6 decimal points in all calculations*. Avoid careless measurements. If your measurements are wrong, repeat them. Don't be afraid to double-check your measurements and your calculations.

Good agreement is defined as: Total momentum before collision = Total momentum after collision $\pm 20\%$

The more carefully you make your angle and gap measurements, the easier it will be to get results within the allowed 20% margin. Careless measurements of the angles or gap lengths can easily put your results outside this margin. Again, don't be afraid to double-check measurements and calculations.

Momentum is defined as the product of mass and velocity. Since velocity is a vector, while mass is a scalar, the product of the two is still a vector; called the momentum vector. The direction of the momentum vector is the same as the direction of the velocity vector. The magnitude of the momentum is the product of the mass and speed.

To add one momentum to another, you must add the momenta as vectors. To start the process, define each vector relative to a coordinate system at its tail. Note: All reference frames must be oriented exactly parallel to each other.



7A. The Component Method. (All magnitudes are in k•gm/s and all angles are in degrees.)

Total momentum before collision = Total momentum after collision

$$mv\angle 0^\circ + MV\angle ? = mv'\angle -|\phi| + MV'\angle \theta$$

?Begin by filling-in the momentum magnitudes and angles corresponding to the terms in the expression above.

Puck#1	Puck#2	=	Puck#1	+	Puck#2
Momentum ₁ before	+ Zero	=	Momentum ₁ after	+	Momentum ₂ after
(_____ \angle _____)	+ 0.0000	=	(_____ \angle _____)	+	(_____ \angle _____)
mv $\angle 0^\circ$	+ 0	=	mv' $\angle - \phi $	+	MV' $\angle \theta$

?Then find the **i** and **j** components of the three momentum vectors and enter these into the equation below.

Puck#1	Puck#2	=	Puck#1	+	Puck#2
Momentum ₁ before	+ Zero	=	Momentum ₁ after	+	Momentum ₂ after
(_____ i + _____ j)	+ (0 i +0 j)	=	(_____ i - _____ j)	+	(_____ i + _____ j)

?Combine the **i** and **j** components on each side separately. Enter the results in the equation below. The two resultant momentum vectors should be equal, or nearly so. If not, find your mistake and correct it before you continue.

Total momentum before collision = Total momentum after collision

$$_____ \mathbf{i} + _____ \mathbf{j} = _____ \mathbf{i} - _____ \mathbf{j}$$

?To complete the analysis, convert these two vectors from component form back into their polar forms.

Total momentum before collision = Total momentum after collision

$$(_____ \angle _____) = (_____ \angle _____)$$

One way to examine the accuracy of this test of momentum conservation is to see how close these two vectors come to canceling each other. This is easiest to do by working with the component forms. Square the difference between the **i**-components. Square the difference between the **j**-components. Add the squared differences together and take the square root. We will calculate the percent difference by taking this result, dividing by the magnitude of the momentum before the collision and converting to a percent difference.

$$\left\{ (| \underline{\hspace{1cm}} | - | \underline{\hspace{1cm}} |)^2 + (| \underline{\hspace{1cm}} | - | \underline{\hspace{1cm}} |)^2 \right\}^{1/2} = \text{magnitude of difference vector} = \underline{\hspace{2cm}}$$

| $\Delta\mathbf{i}$ -components| | $\Delta\mathbf{j}$ -components|

%Difference = 100% x (magnitude of difference vector) / **mv** = _____ % (must be less than 20%)

7B. The Vector Triangle Method. Law of Sines I

Here you will test momentum conservation by checking the momentum vector triangle using the Law of Sines. Slide the vectors, without changing their directions, until they form a vector triangle. Draw the vector triangle showing the initial and final momentum vectors of the two pucks. The vectors after collision are added head-to-tail. The initial momentum vector of puck #1 should be drawn to show that it is the sum of the other two vectors.

The outline for this vector triangle is drawn for you below. It is up to you to show the direction of the arrows representing the vectors; label the length of the sides with the appropriate symbols, namely $m\mathbf{v}$, $m\mathbf{v}'$, and $M\mathbf{V}'$; and **label all the interior and exterior angles** on this triangle (not with numbers but with symbols). Identify the angles in terms of the measured angles, θ , and $|\phi|$. (You may also use the 180° angle, in the symbolic labels.)

Let $m\mathbf{v}$ = side a = _____ kg·m/s

Let $m\mathbf{v}'$ = side b = _____ kg·m/s

Let $M\mathbf{V}'$ = side c = _____ kg·m/s

Then

A is the angle opposite side a

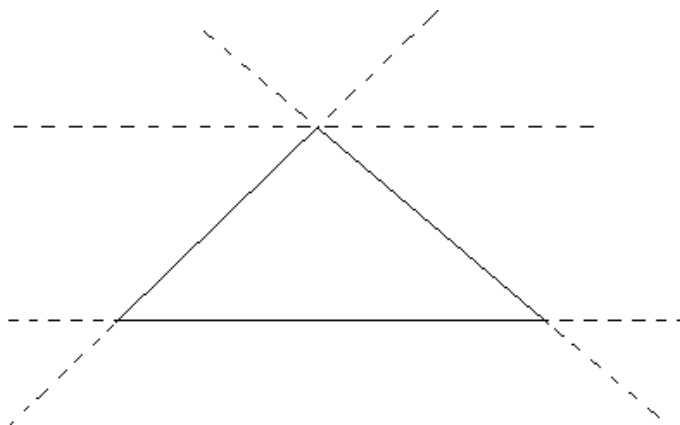
A = _____[°]

B is the angle opposite side b

B = _____[°]

C is the angle opposite side c

C = _____[°]



(Label all sides, interior angles and exterior angles of this triangle with Symbols.)

Now check your solution to see how well the law of sines holds for this triangle. Show below both the symbolic equivalents and the numeric results from the law of sines. (a, b, and c are the sides; A, B, and C are the angles.)

$$a / \sin A = b / \sin B = c / \sin C$$

Write the Law of Sines Equation in terms of the symbols 180° , $m\mathbf{v}$, $m\mathbf{v}'$, $M\mathbf{V}'$, θ , and $|\phi|$:

(a / sin A =) _____ = (b / sin B =) _____ = (c / sin C =) _____

Evaluate the expressions above using your experimental values and record the numeric results below.

(a / sin A =) _____ = (b / sin B =) _____ = (c / sin C =) _____

If the numbers in the row above are close together that is a good sign, but we need to check this solution further. In the next section you will use the Law of Sines to solve for each of the angles, A, B and C. You will use the internal angles of the triangle to calculate θ , and $|\phi|$, and then compare the calculated and measured angles.

7C. The Vector Triangle Method. Law of Sines II

To see how close your solution comes to verifying the conservation of momentum, check to see if the law of sines can correctly predict the angles of the triangle. Show the symbolic and numeric equivalents from the law of sines.

$$a / \sin A = b / \sin B = c / \sin C$$

Again let the sides be $a = m\mathbf{v}$, let $b = m\mathbf{v}'$, and let $c = M\mathbf{V}'$. Label these sides on your vector triangle if you haven't already done so. Identify and label the angles A, B, and C opposite these sides on the vector triangle. Now solve for the angles A, B and C both symbolically and numerically. Refer to the diagram on the previous page to get the appropriate symbolic equivalents. *(Use your measured angles and momenta in the numeric evaluation. Use the arcsine or \sin^{-1} function to solve for the angles. If you end up trying to take the arcsine of a number greater than 1.0000, your answer can only be 90° . If the expected angle is greater than 90° , the answer on your calculator must be adjusted by hand because the calculator only gives answers between $+90^\circ$ and -90° .)*

Expression in terms of symbols

a, b, B

Expression in terms of symbols

$m\mathbf{v}$, $m\mathbf{v}'$, and θ .

Numeric value

What angle were you expecting (from your vector diagram)?

$$A = \sin^{-1}(\underline{\hspace{2cm}}) = \sin^{-1}(\underline{\hspace{2cm}}) = \underline{\hspace{1cm}}^\circ = \underline{\hspace{1cm}}^\circ$$

Expression in terms of symbols

b, c, C

Expression in terms of symbols

$m\mathbf{v}'$, $M\mathbf{V}'$, and $|\phi|$.

Numeric value

What angle were you expecting (from your vector diagram)?

$$B = \sin^{-1}(\underline{\hspace{2cm}}) = \sin^{-1}(\underline{\hspace{2cm}}) = \underline{\hspace{1cm}}^\circ = \underline{\hspace{1cm}}^\circ$$

Expression in terms of symbols

c, a, A

Expression in terms of symbols

$M\mathbf{V}'$, $m\mathbf{v}$, and θ & $|\phi|$.

Numeric value

What angle were you expecting (from your vector diagram)?

$$C = \sin^{-1}(\underline{\hspace{2cm}}) = \sin^{-1}(\underline{\hspace{2cm}}) = \underline{\hspace{1cm}}^\circ = \underline{\hspace{1cm}}^\circ$$

Add the angles to see if your triangles has 180° . Sum of Angles = $A + B + C = \underline{\hspace{2cm}}^\circ$

7D. The Vector Triangle Method. Law of Cosines

Let's try a similar check using the Law of Cosines, this time, to predict the magnitudes of the momentum vectors. First, **write them all symbolically**. Use the same symbols as above: let $a = \mathbf{mv}$, let $b = \mathbf{mv}'$, and let $c = \mathbf{MV}'$ and angle A is opposite side a, angle B is opposite side b, and angle C is opposite side c. Then evaluate each expression using the numeric values from your own collision measurements of \mathbf{mv} , \mathbf{mv}' , \mathbf{MV}' , θ , and $|\phi|$.

$$a^2 = b^2 + c^2 - 2bc \cos A = \underline{\hspace{15cm}}$$

$$a^2 = \underline{\hspace{15cm}} = \underline{\hspace{15cm}}$$

Therefore, the calculated value of a is: $a = \mathbf{mv} = \underline{\hspace{5cm}}$ kg·m/sec %Error in a = $\underline{\hspace{2cm}}$ %
(Assume the measured value of \mathbf{mv} is the correct one when calculating the %Error in this case.)

$$b^2 = c^2 + a^2 - 2ca \cos B = \underline{\hspace{15cm}}$$

$$b^2 = \underline{\hspace{15cm}} = \underline{\hspace{15cm}}$$

Therefore, the calculated value of b is: $b = \mathbf{mv}' = \underline{\hspace{5cm}}$ kg·m/sec %Error in b = $\underline{\hspace{2cm}}$ %
(Assume the measured value of \mathbf{mv}' is the correct one when calculating the %Error in this case.)

$$c^2 = a^2 + b^2 - 2ab \cos C = \underline{\hspace{15cm}}$$

$$c^2 = \underline{\hspace{15cm}} = \underline{\hspace{15cm}}$$

Therefore, the calculated value of c is: $c = \mathbf{MV}' = \underline{\hspace{5cm}}$ kg·m/sec %Error in c = $\underline{\hspace{2cm}}$ %
(Assume the measured value of \mathbf{MV}' is the correct one when calculating the %Error in this case.)

8. Elastic and Inelastic Collisions:

When objects collide, some of the kinetic energy in their motion ends up as internal heating, solid vibrations, noise, or some other form of internal or external energy. If the objects stick together, the collision is called completely inelastic. Totally elastic collisions, on the other hand, conserve kinetic energy, but they only happen at the atomic level. A bouncing superball, by comparison, experiences a collision with the floor that is about 80% elastic. Billiard balls collide with each other in collisions that are nearly 100% elastic. Calculate the kinetic energy before and after the collision to see how well energy is conserved in your case.

Use the known masses and the measured velocities from Section 4 on Page 2.

Analysis of Elasticity of Collision:

Before the collision: $K_{mv} = \underline{\hspace{2cm}} \text{ J} + K_{MV} = \underline{\hspace{2cm}} 0.00 \text{ J} = \Sigma K_{\text{Before}} = \underline{\hspace{2cm}} \text{ J}$

After the collision: $K_{mv'} = \underline{\hspace{2cm}} \text{ J} + K_{MV'} = \underline{\hspace{2cm}} \text{ J} = \Sigma K_{\text{After}'} = \underline{\hspace{2cm}} \text{ J}$

For your collision, report the "%Elasticity of collision" as $[100\% \times \Sigma K_{\text{After}'} / \Sigma K_{\text{Before}}]$.

%Elasticity of collision = $\underline{\hspace{2cm}}$ % (The correct answer must be less than 100%)

Where did the missing energy go? (Make a list of three ways in which energy escapes from the colliding pucks. You cannot use friction. That one has already been given to you. Find three of your own. We will assume that, during the brief time interval from an instant just before the collision to an instant just after the collision, no significant amount of energy is lost to friction. The closer we make our measurements to the point of impact the better this assumption becomes. But in that instant the energy went somewhere. Name three channels for energy loss during the collision.)

1.

2.

3.

9. Worksheet and Reference Page:

Copy the important intermediate results here as you proceed through the calculations. This is for your benefit. It is too easy to get confused when you need to keep looking through the earlier pages for the important numbers to use them in your later calculations.

Keep this page filled in with current numbers and keep it with you as you make your calculations. This will make your life easier. Show all the numbers on this page as though they have **6 significant digits**. That minimizes rounding errors and keeps the calculations as precise as possible at every step.

$$m = \text{_____ kg}$$

$$v = \text{_____ m/s}$$

$$mv = a = \text{_____ kg}\cdot\text{m/s}$$

$$v' = \text{_____ m/s}$$

$$mv' = b = \text{_____ kg}\cdot\text{m/s}$$

$$M = \text{_____ kg}$$

$$V' = \text{_____ m/s}$$

$$MV' = c = \text{_____ kg}\cdot\text{m/s}$$

$$180 - \theta - |\phi| = A = \text{_____}^\circ$$

$$\theta = B = \text{_____}^\circ$$

$$|\phi| = C = \text{_____}^\circ$$

Trigonometric functions used in calculations

(For vector component AND vector triangle calculations)

$$\sin \theta = \text{_____} \quad \cos \theta = \text{_____}$$

(For vector component calculations, only)

$$\sin -|\phi| = \text{_____} \quad \cos -|\phi| = \text{_____}$$

(For vector triangle calculations, only)

$$\sin |\phi| = \text{_____} \quad \cos |\phi| = \text{_____}$$

$$\sin (180 - \theta - |\phi|) = \text{_____} \quad \cos (180 - \theta - |\phi|) = \text{_____}$$

Questions: (Review of vector addition) Use a separate piece of paper if your neat & complete answers won't fit in the space provided here. **Draw and label a diagram for each case.**

1. Add these vectors and express each result in polar form.

a) $34 \text{ kg}\cdot\text{m}/\text{sec} \angle 22^\circ + 86 \text{ kg}\cdot\text{m}/\text{sec} \angle 86^\circ$

b) $421 \text{ m}/\text{sec} \angle 0^\circ + 302 \text{ m}/\text{sec} \angle 192^\circ$

2. Subtract these vectors and express each result in polar form.

a) $45 \text{ newtons} \angle 120^\circ - 62 \text{ newtons} \angle -133^\circ$

b) $82.3 \text{ meters} \angle 160^\circ - 89.1 \text{ meters} \angle 70^\circ$

3. Find the vector that will cancel out the sum of these two vectors. Convert the result to polar form.

$33 \angle 0^\circ + 41 \angle 140^\circ$.