Name: ______________________________ **Book:** ___ **Period:** ___ **Due Date:** _________
Lab Partners: ____

FINDING the *f*OCAL LENGTHS of LENSES

Purpose: To explore simple techniques for using the thin lens equation to estimate the focal lengths of converging and diverging lenses.

Converging Lenses

Lens Set: ___________

Part I: Estimating focal length - Method I (Point-Focus Estimate)

1. Use a distant light source as the object (light bulb). Assume the object distance is infinite.
2. Measure the image distance using a 3x5 card as a screen to locate the image.
3. We know $1/i + 1/o = 1/f$. We assume o is infinite, so $1/o = 0$, thus $1/i = 1/f$, and $i = f$.

Lens A f_A = _______________ cm — Long Focal Length Lens

Lens B f_B = _____________ cm — Short Focal Length Lens

Analysis I:

Testing the Assumption:

We assumed the object was at infinity so that we could estimate the focal length by setting it equal to the image distance (assumed $i = f$). Obviously, this assumption is not true when the object distance is less than infinity. What we will test here are the consequences of that faulty assumption, i.e. how much error does it introduce into our estimate the focal length. Imagine performing this test on a lens with a true focal length of 25.000 cm. Use the thin lens equation to calculate the image distance at each of the specified object distances in the table. You will get a number that is slightly greater than 25.000 cm for all object distances less than infinity. The image distance **is** your estimated focal length. Calculate the %Error between the estimated focal length and the true focal length.

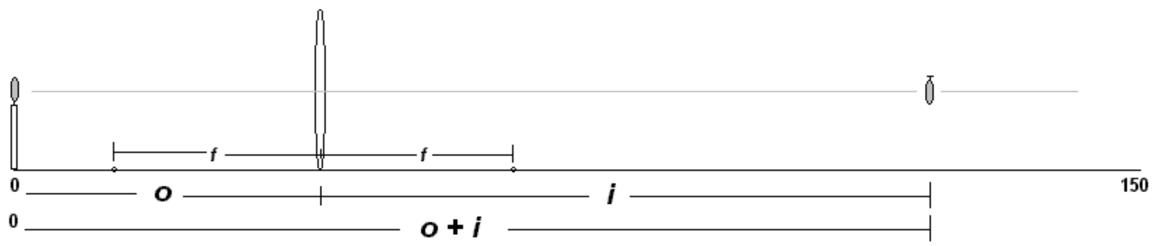
Object Distance	Image Distance	Estimated Focal Length	%Error (True f = 25cm)
10 m = 1,000 cm	_________	_________	_________
30 m = 3,000 cm	_________	_________	_________
50 m = 5,000 cm	_________	_________	_________
100 m = 10,000 cm	_________	_________	_________

To get 0.1%Error, what minimum object distance do you need? ________ m

Part II: Estimating focal length - Method II – Lens A

1. Use a small candle as the object and a screen to locate the real image of the flame, as follows:
2. (a) <u>Keep the candle at the zero position</u>. (b) Record the object distance. (c) Hold the lens at the desired object distance, $\boldsymbol{o}$. (d) Move the screen until the focused image is found. (e) Record $\boldsymbol{o} + \boldsymbol{i}$.
3. Select object distances ranging from approximately <u>1.5 times</u> the focal length (see Method I) to approximately <u>5 times</u> the focal length. If the (5•$\boldsymbol{f}$) distance from the candle to the screen, $\boldsymbol{o} + \boldsymbol{i}$, exceeds 150 cm, set the screen at 150 cm and adjust the lens position to find the $\boldsymbol{o}$ nearest to 5•$\boldsymbol{f}$, instead. (Best data between 1.5•$\boldsymbol{f}$ and 4•$\boldsymbol{f}$.)

<u>Sample set-up with the candle at the minimum recommended object distance</u>. This is the setup for the last line in the Data Table. The candle is always at zero. Set the lens on an exact whole centimeter near 1.5•$\boldsymbol{f}$. Do not exceed 150 cm between the candle and the screen ($\boldsymbol{o} + \boldsymbol{i}$) on any setup. <u>Use whole numbers for object distances</u> on all set-ups. Record all screen ($\boldsymbol{o} + \boldsymbol{i}$) positions to the nearest 0.1 cm.



Data & Results Table: Lens A: $\boldsymbol{f}_A$ (Estimated from Part I) = ____________ cm

|-------------- Data- ------------| |------------ See Analysis III- --------------|

	$\boldsymbol{o}$ (cm) (Candle to Lens)	$\boldsymbol{o} + \boldsymbol{i}$ (cm) (Candle to Screen)	$\boldsymbol{i}$ (cm) (Lens to Screen)	$\boldsymbol{f}_A$ (cm) (Lens to Focal Point, use Thin Lens Equation)	
(~5.0 • f) =	____________	____________	____________	____________	
(~4.0 • f) =	____________	____________	____________	____________	
(~3.5 • f) =	____________	____________	____________	____________	
(~3.0 • f) =	____________	____________	____________	____________	
(~2.75 • f) =	____________	____________	____________	____________	
(~2.5 • f) =	____________	____________	____________	____________	
(~2.25 • f) =	____________	____________	____________	____________	
(~2.0 • f) =	____________	____________	____________	____________	(Make sure $\boldsymbol{o} + \boldsymbol{i} = 2\boldsymbol{o}$)
(~1.5 • f) =	____________	____________	____________	____________	(Make sure $\boldsymbol{o} + \boldsymbol{i} = 3\boldsymbol{o}$)

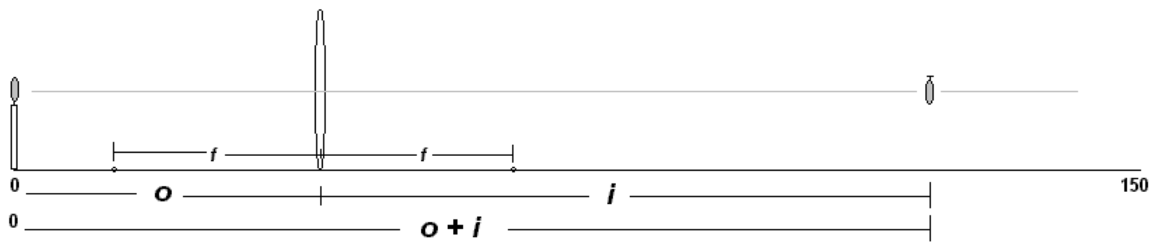
Average focal length

f_A = ____________ cm

Part III: Estimating focal length - Method II – Lens B

1. Use a small candle as the object and a screen to locate the real image of the flame, as follows:
2. (a) Keep the candle at the zero position. (b) Record the object distance. (c) Hold the lens at the desired object distance, $\boldsymbol{o}$. (d) Move the screen until the focused image is found. (e) Record $\boldsymbol{o} + \boldsymbol{i}$.
3. Select object distances ranging from approximately 1.5 times the focal length (see Method I) to approximately 5 times the focal length. If the ($5{\bullet}f$) distance from the candle to the screen, $\boldsymbol{o} + \boldsymbol{i}$, exceeds 150 cm, set the screen at 150 cm and adjust the lens position to find the $\boldsymbol{o}$ nearest to $5{\bullet}f$, instead. (Best data between $1.5{\bullet}f$ and $4{\bullet}f$.)

Sample set-up with the candle at the minimum recommended object distance. This is the setup for the last line in the Data Table. The candle is always at zero. Set the lens on an exact whole centimeter near $1.5{\bullet}f$. Do not exceed 150 cm between the candle and the screen ($\boldsymbol{o} + \boldsymbol{i}$) on any setup. Use whole numbers for object distances on all set-ups. Record all screen positions ($\boldsymbol{o} + \boldsymbol{i}$) to the nearest 0.1 cm.



Data & Results Table: Lens B:

f_B (Estimated from Part I) = ____________ cm

	------------- Data- ------------		------------ See Analysis III- -------------	
	$\boldsymbol{o}$ (cm) (Candle to Lens)	$\boldsymbol{o} + \boldsymbol{i}$ (cm) (Candle to Screen)	$\boldsymbol{i}$ (cm) (Lens to Screen)	f_B (cm) (Lens to Focal Point, use Thin Lens Equation)
($\sim 5.0 \bullet f$) =	____________	____________	____________	____________
($\sim 4.0 \bullet f$) =	____________	____________	____________	____________
($\sim 3.5 \bullet f$) =	____________	____________	____________	____________
($\sim 3.0 \bullet f$) =	____________	____________	____________	____________
($\sim 2.75 \bullet f$) =	____________	____________	____________	____________
($\sim 2.5 \bullet f$) =	____________	____________	____________	____________
($\sim 2.25 \bullet f$) =	____________	____________	____________	____________
($\sim 2.0 \bullet f$) =	____________	____________	____________	____________ (Make sure $\boldsymbol{o} + \boldsymbol{i} = 2\boldsymbol{o}$)
($\sim 1.5 \bullet f$) =	____________	____________	____________	____________ (Make sure $\boldsymbol{o} + \boldsymbol{i} = 3\boldsymbol{o}$)

Average focal length

f_B = ____________ cm

Analysis II & III: (Experience suggests using one ***LP3*** page for Lens A and another page for Lens B.)

a. Average Values of Focal Length

Use the thin lens equation to calculate the apparent focal length at each object distance in the table. Then calculate the average focal length from these results. List your calculated average focal lengths in the Data Tables and recopy them here.

Average focal length: Lens A: f_A = __________ cm Lens B: f_B = __________ cm

Estimating the Error in an Average

For **N** independent measurements of an experimental variable, taking the average (the "*sample average*") may seem like an obvious way to estimate the true value of the variable. Just add up the values and divide by **N**, the number of values in the sample. What is not obvious is how close a particular sample average is to the true value (called the "*population average*" = the average of an infinite number of independent measurements.). To gauge how close the sample average is to the population average we need to know the standard deviation (the width of the population of measurements). This section will provide only a very simple introduction to the standard deviation and how to interpret it.

The standard deviation is related to the probability that the population average falls within one standard deviation of a given measurement. Assuming independent measurements for which the errors are truly random, then the population average (*the true value of a variable*) will fall within one standard deviation of a measurement 68.26% of the time. The population average falls within two standard deviations of a measurement 95.45% of the time. Moreover, the population average falls within three standard deviations of a measurement 99.73% of the time. If only we knew the population standard deviation we would be all set.

From a finite set of measurements, however, we can estimate the standard deviation for the population of all possible measurements. We estimate the population standard deviation using the sample variance, $\mathbf{s}_N^2$:

$$\sigma_N \sim \mathbf{s}_N = \text{sqrt}\left\{[\Sigma(x_i - x_{AVE})^2] / \mathbf{N}\right\}$$

Since the data is constrained to center around the average, the population standard deviation is often estimated using the square root of the bias corrected sample variance, $\mathbf{s}_{N-1}^2$, as follows:

$$\sigma_{N-1} \sim \mathbf{s}_{N-1} = \text{sqrt}\left\{[\Sigma(x_i - x_{AVE})^2] / [\mathbf{N} - 1]\right\}$$

You will use this later definition as our estimate of the standard deviation, σ, in your analysis. Report your average focal lengths with ±95% confidence intervals. The 95% confidence interval equals $1.95995 \cdot \sigma_{N-1}$.

FOR Lens A

Standard deviation = σ_{N-1} = ________________ cm; f_A = _______________ cm ± _________________ cm

Average goes here 95% confidence interval goes here

FOR Lens B

Standard deviation = σ_{N-1} = ________________ cm; f_B = _______________ cm ± _________________ cm

Average goes here 95% confidence interval goes here

Repeating these measurements on the same lenses should produce estimates of focal length that are within the specified interval 95% of the time. If the errors are truly random, then the confidence limits on those future experiments should be about the same as these.

b. Graph 1A and Graph 1B – One graph for each lens – (*i* vs *o*).

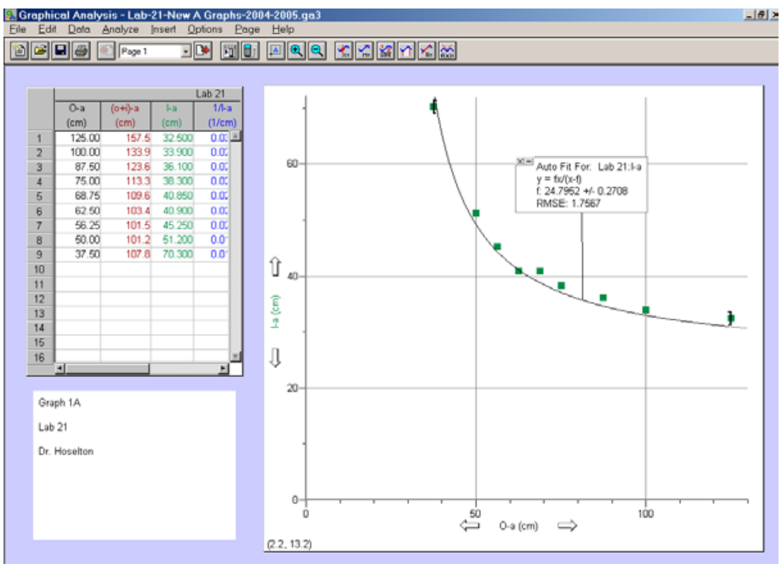
b. i. Use the two manual columns supplied by ***LP3*** to store values of ***i*** and ***o*** for lens A. Create two more manual columns to hold ***i*** and ***o*** for lens B. In each graph put the image distance on the y-axis and the object distance on the x-axis (***i*** vs ***o***). Put your name, the Lab Number, and the Graph number in the text box.

b. ii. Find the best fit for the data points using a rearranged version of the thin lens equation.

$$i = o\,f / (o - f)$$

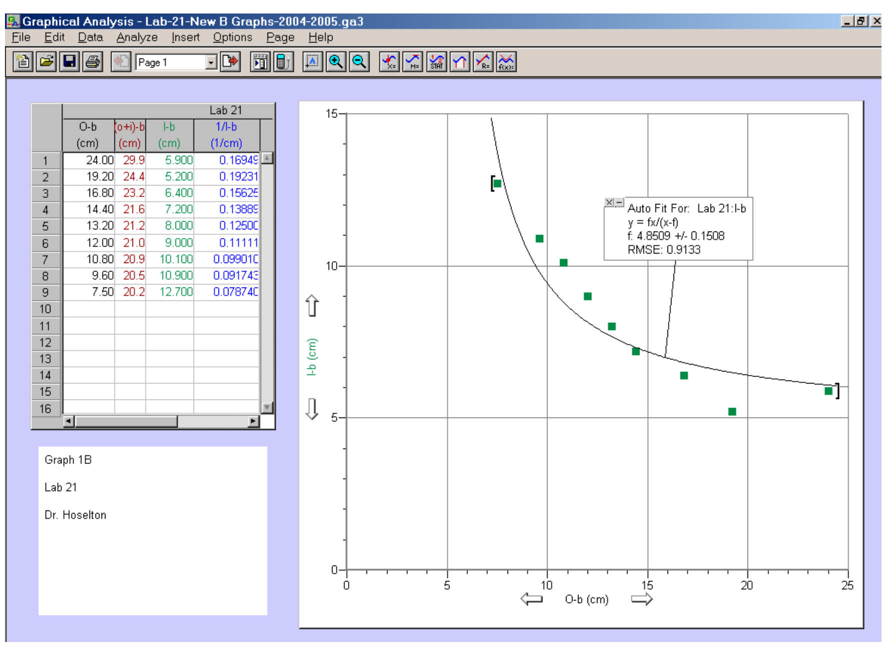
In the equation box of ***LoggerPRO3*** enter:	***f****x/(x–***f***)

Graph 1A: Lens A: f_A = ___________ cm   Graph 1B: Lens B: f_B = ___________ cm



Make sure that the interval for the fit, as shown by the two square brackets, [], includes all the data.

Check the brackets on every graph before you print it. If the brackets do not include all the data points, you can use your mouse to click and slide each bracket into the correct location.



c. Graph 2A and Graph 2B – One graph for each lens – (1/$\boldsymbol{i}$ vs 1/$\boldsymbol{o}$).

c. i. Create new calculated columns in ***LP3*** to calculate the inverses of $\boldsymbol{i}$ and $\boldsymbol{o}$ for each lens. Put the inverse of the image distance on the y-axis and the inverse of the object distance on the x-axis (1/$\boldsymbol{i}$ vs 1/ $\boldsymbol{o}$). Put your name, the Lab number, and the Graph number in the text box.

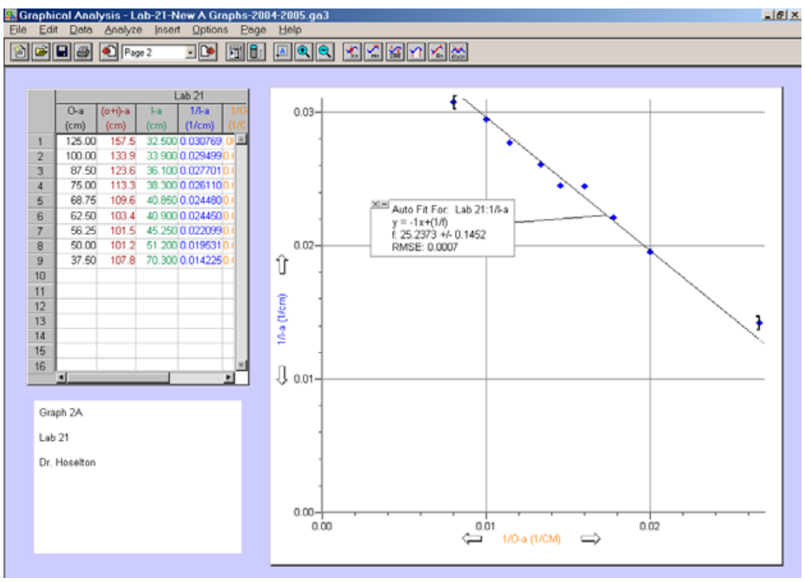
c. ii. Find the best fit for the data points using a rearranged version of the thin lens equation.

$$1/\boldsymbol{i} = -1/\boldsymbol{o} + 1/\boldsymbol{f}$$

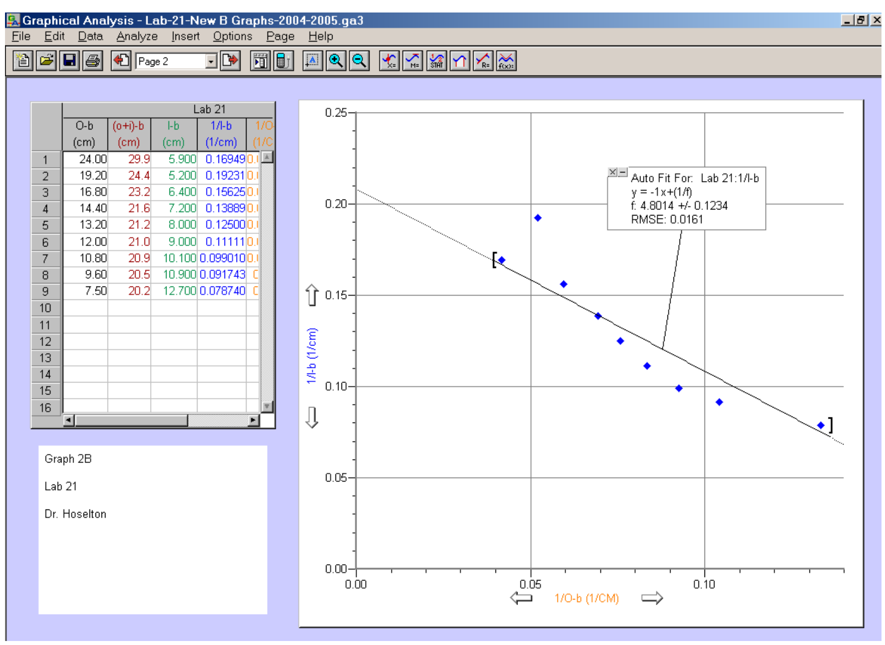
This should produce a straight line with a slope of minus one and an intercept of 1/$\boldsymbol{f}$.

In the define function box of ***LoggerPRO3*** enter:	$-1*x+(1/f)$

Graph 2A: Lens A: f_A = ___________ cm   Graph 2B: Lens B: f_B = __________ cm



Note that the brackets in both of these graphs correctly include all the data points.



d. Graph 3A and Graph 3B – One graph for each lens – ($\boldsymbol{i}$ vs. 1+($\boldsymbol{i}$ /$\boldsymbol{o}$)).

d. i. Create new calculated columns in ***LP3*** to calculate 1+($\boldsymbol{i}$ /$\boldsymbol{o}$) for each lens. Put the image distance on the y-axis. Put 1+($\boldsymbol{i}$ /$\boldsymbol{o}$) on the x-axis. Create a graph of Image Distance vs 1+($\boldsymbol{i}$ /$\boldsymbol{o}$) for each lens from the appropriate Data Table. Put your name, the Lab Number, and the Graph number in the text box.

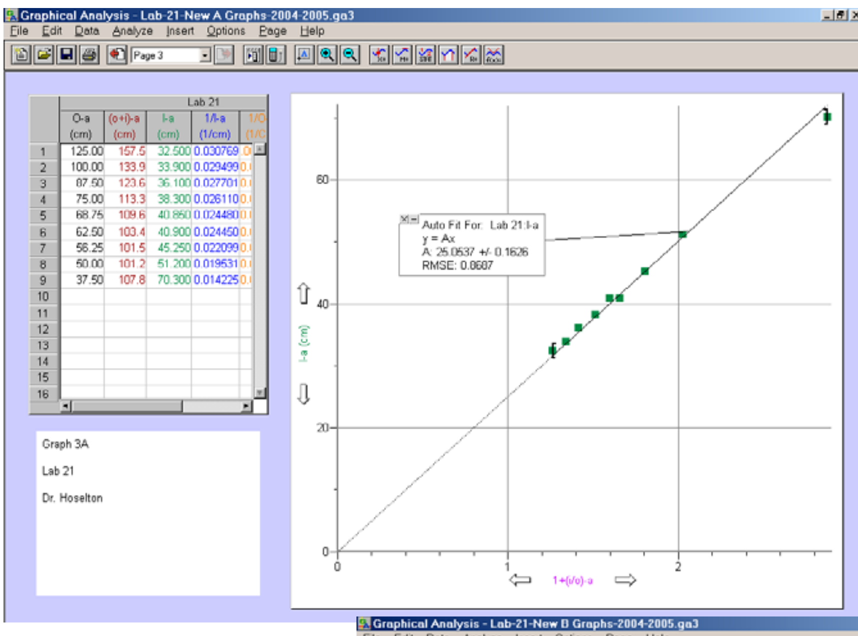
d. ii. Find the best fit for the data points using a rearranged version of the thin lens equation.

$$\boldsymbol{i} = \boldsymbol{f}\,[1 + (\boldsymbol{i}/\boldsymbol{o})]$$

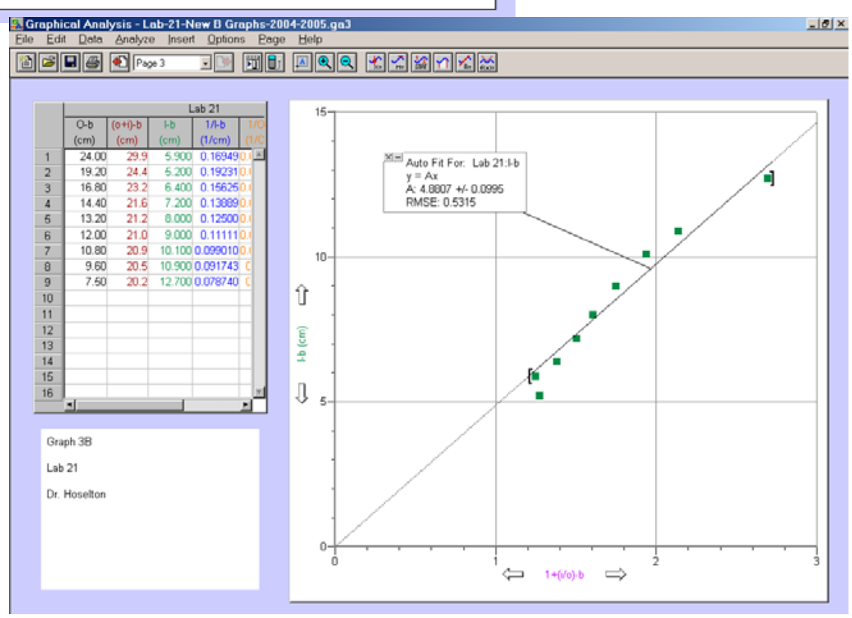
This should produce a straight line through the origin with a slope of $\boldsymbol{f}$.

In the equation box of ***LoggerPRO3*** enter:	$\boldsymbol{f}$*x

Graph 3A: Lens A: f_A = ___________ cm Graph 3B: Lens B: f_B = ___________ cm



Note that both of these graphs correctly include all the data points.



Final Note on Graphs: Before you print the graphs, create a seventh page in ***LP3***. As always copy the current page. Then delete the Graph. In the text box on page 7 include just your name and Lab #. Expand the Data Table so that all the columns are visible at the same time. Adjust their widths if necessary to make them all fit and still be able to read all the numbers. When you print the graphs print out all seven pages including the page with the Data Table in **LANDSCAPE** mode.

Question:

Which analytic technique(s), when applied to the data obtained in this lab will, in your opinion, produce an estimate of the focal length more accurate than the estimate obtained from the Point-Focus Method in Part I?

Chose among these four methods:

(a)-Average f from many measurements, and/or
(b)-Graph 1, and/or
(c)-Graph 2, and/or
(d)-Graph 3.
One, two, three or all four of these methods might yield a more accurate estimate of the focal length than Method I. Defend your answer(s) in terms of procedure rather than in terms of your personal results.

Answer: ________________________________

Conclusion: ______________________________

____

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____

____

Use the remainder of this page to complete your analysis and justify you Answer and Conclusion.

Diverging Lenses

Part IV: Estimating Focal Length – Method III – Diverging Lens

The focal length of a diverging lens cannot be measured directly by optical means. A diverging lens has a negative focal length and will not form a real image. Combining a diverging lens with a stronger converging lens, however, will give a real image. When two lenses are placed very close to each other, they have a combined focal length that can be calculated using

$$1 / f_C = 1 / f_1 + 1 / f_2$$

where f_1 is the focal length of the converging lens, f_2 is the focal length of the diverging lens, and f_C is the focal length of the combined lens (*created by holding the two lenses close together face to face*). This method works for any pair of lenses: two converging lenses, two diverging lenses, or one of each type.

Procedure:

1. Obtain a new short focal length converging lens, if necessary, from your instructor and measure its focal length (f_1) using Methods I and II. Measure 4 points by Method II and find the average focal length.

   If your original short focal length lens is strong enough, your instructor will allow you to use it again here. In that case, reenter the appropriate data from the Data & Results Table for Lens B into the table below, including your estimate for lens B using Method I. In this section lens B is known as lens 1.

   Average the four estimates of the focal length obtained using Method II and enter the average in the box at the end of this table.

Data Table: Lens 1:

f_1 (Estimated using Method I) = ____________ cm

	|- - - - - - - - - - - - - - Data- - - - - - - - - - - - -|			
	o (cm) (Candle to Lens)	***o + i*** (cm) (Candle to Screen)	***i*** (cm) (Lens to Screen)	f_1 (cm) (Lens to Focal Point, use Thin Lens Equation)
($4 \bullet f_1$) =	____________	____________	____________	____________
($3 \bullet f_1$) =	____________	____________	____________	____________
($2.5 \bullet f_1$) =	____________	____________	____________	____________
($2 \bullet f_1$) =	____________	____________	____________	____________

Average focal length
f_1 = __________ cm

2. Combine a short focal length converging lens with a longer focal length diverging lens. The lenses held face-to-face form a converging lens. Make sure you hold them together firmly face-to-face while you collect the following data. Measure the focal length of the combined lens (f_C) using Methods I and II. Measure 4 points by Method II and enter the data in the table below.

   Average the four estimates for the focal length obtained using Method II and enter the average in the box at the end of this table.

Data Table: Combined Lens:

f_C (Estimated using Method I) = ____________ cm

|- - - - - - - - - - - - - - Data- - - - - - - - - - - - -|

	o (cm) (Candle to Lens)	$o + i$ (cm) (Candle to Screen)	i (cm) (Lens to Screen)	f_C (cm) (Lens to Focal Point, use Thin Lens Equation)
$(4 \bullet f_C)$ =				
$(3 \bullet f_C)$ =				
$(2.5 \bullet f_C)$ =				
$(2 \bullet f_C)$ =				

Average focal length

f_C = ___________ cm

Analysis IV:

Focal lengths estimated using Method I and Method II should be similar in both cases. Correct any problems before you attempt this analysis.

Use the average focal length of Lens 1 and the average focal length of the combined lens to calculate the focal length of the diverging lens (f_2) using the equation at the top of page 9. Note that f_2 will be negative, probably between about –10 cm and –20 cm. The focal length, f_2, is expected to be negative since lens 2 is a diverging lens.

f_2 = _______________ cm

Assume you have a diverging lens with a focal length of –12 cm. Find the apparent focal length of the combinations of that diverging lens with each of the following converging lenses. Indicate with circles which combinations would be **Useful** and which are **NOT Useful** for finding f_2, the focal length of the diverging lens?

1. f_1 = +15 cm f_2 = –12 cm f_C = __________________ cm **Useful / NOT Useful**
2. f_1 = +12 cm f_2 = –12 cm f_C = __________________ cm **Useful / NOT Useful**
3. f_1 = +10 cm f_2 = –12 cm f_C = __________________ cm **Useful / NOT Useful**
4. f_1 = + 6 cm f_2 = –12 cm f_C = __________________ cm **Useful / NOT Useful**

Questions for Analysis:

1. Complete the following table. Each column represents a different lens. Assume all measurements are in centimeters. Lower-case "***f***" stands for ***focal length***. Lower-case "***i***" stands for ***image distance.*** Lower-case "***o***" stands for ***object distance.*** Upper-case "**M**" stands for ***magnification***. When there is no + or – in front of a number, you must supply one.

   You may use CON and DIV as shorthand for Converging and Diverging in this table.

	a	b	c	d	e	f	g	h
Type	Convrgng				Diverging		Diverging	
f (cm)	_20		–20		–40	–20		
i (cm)					–10		_40	
o (cm)	+10	+100	+30	+60				+24
M = -i/o		–2		-0.5		+0.10	+0.5	_0.50
Real Image?		yes					no	
Upright Image?								no

2. **Challenge -** This question is **not** optional. Show that the thin-lens equation can be rewritten as

$$f^2 = x\,x'$$

   Where x is the distance from the object to the nearest focal point and x' is the distance from the image to its nearest focal point. The ***algebraic derivation*** of this equation starts with the thin lens equation, makes a couple of substitutions, and solves the equation for f^2. This involves only simple algebraic manipulations of the equation. (Hint: *draw a diagram to visualize it.*)

   If, instead, you wish to try a ***geometric derivation*** using similar triangles, make sure your diagram is especially-clearly labeled, and the steps clearly annotated, so your derivation is easily readable. (*Show the entire derivation on this page*.)

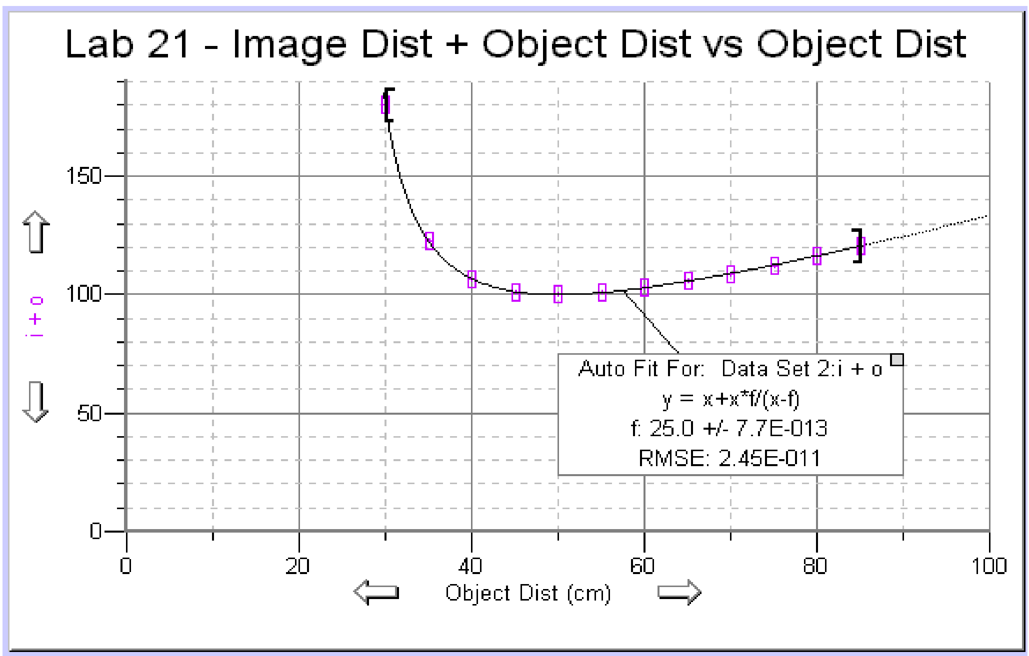
Commentary on Graph 3.

For those who are interested in the function that describes $(o + i)$ as a function of o, here is a brief analysis and a sample graph for a hypothetical 25 cm focal length lens.

While taking the data for Part II and Part III, some students notice that the function is non-linear. In fact, it has a minimum at $o = 2.0 \cdot f$ and $(o + i) = 4.0 \cdot f$. It rises steeply from this minimum at shorter object distances, and rises less steeply from this minimum at longer object distances. We can derive the function that describes $o + i$ as a function of o, by applying the Thin Lens Equation. The result is

$$(o + i) = o + [o\,f / (o - f)]$$

A plot of $(o + i)$ vs o can be fit with this function using ***LoggerPRO3***. Synthetic data and the graphical fit are shown on the graph below.



If you set the derivative of this function equal to zero and evaluate, you will find, at the minimum of the graph, that

$$o_{\text{at the minimum}} = 2 \cdot f$$

This agrees with what can be seen on the graph where the focal length is $f = 25$ cm and the minimum occurs near $o = 50$ cm. Thus, $o_{\text{at the minimum}} = 2 \cdot f$ seems entirely reasonable. Maybe this calculus stuff works after all!

FOR EXTRA CREDIT: students with an interest are invited to derive the equation for this function, graph their own data for Lens A, and then fit the data with this function.

Finally, note that for each value of $(o + i)$ there are two values for o (*except at the minimum*). One way to gather data for a lens is to set the candle and the screen a fixed distance apart and then find the two lens locations where a focused image appears on the screen. The only stipulation is that $(o + i) > 4 \cdot f$.