

Name: _____ Period: _____ Due Date: _____
 Lab Partners: _____

P ENDULUM M OTION

Purpose: To explore the behaviors of a simple pendulum and a physical pendulum. Gravity provides a restoring force to the pendulum analogous to the restoring force of a spring. Like the spring system, the pendulum will oscillate about its equilibrium position and we can use the harmonic motion of the pendulum to find the proportionality constant between the restoring torque and the displacement.

Theory: The Simple Pendulum. Our Analysis of the Simple Harmonic Motion of a spring produced this equation for the period of oscillation,

$$T = 2\pi (m/k)^{1/2},$$

Where **m** is the mass of the system and **k** is the proportionality constant of the Hooke's Law restoring force.

A pendulum consists of a mass moving along the arc of a vertical circle. Thus, the equation for the restoring force of a spring, $F = k \Delta x$, must be replaced by an equation for the restoring torque of the an object in circular motion. By analogy, the restoring torque of a pendulum is given by, $\tau = k \Delta\theta$, where $\Delta\theta$ is the angle of rotation, in radians, of the pendulum away from its equilibrium position at the bottom of the arc. The equation for the period of any pendulum is, by analogy,

$$T = 2\pi (I/k)^{1/2} = 2\pi (I \Delta\theta / \tau)^{1/2} = 2\pi [(I)(\Delta\theta)/(L \cdot mg \sin \Delta\theta)]^{1/2}$$

Where we've used the definition of torque to replace τ with $L \cdot mg \sin \Delta\theta$. For small angles expressed in radians we can also use an important trigonometric approximation; namely, $\sin \Delta\theta \sim \Delta\theta$. Therefore,

$$T = 2\pi [(I)(\Delta\theta)/(L \cdot mg \Delta\theta)]^{1/2} = 2\pi [(I)/(L \cdot mg)]^{1/2}$$

If we are talking about a simple pendulum, **L** is the length of the pendulum and $I = mL^2$. Therefore,

$$T = 2\pi [(mL^2)/(L \cdot mg)]^{1/2} = 2\pi [(mL)/(mg)]^{1/2} = 2\pi [L/g]^{1/2}$$

This is our first major result; a model for the period of a simple pendulum.

$$T = 2\pi \sqrt{\frac{L}{g}} \quad \text{thus} \quad T^2 = 4\pi^2 \frac{L}{g}$$

Equation 1

Because we have invoked the approximation that $\sin \Delta\theta \sim \Delta\theta$, our model is thereby limited to small angular displacements. Experimentally, you are obligated to make sure the displacements are kept small.

Theory: The Physical Pendulum. In the case of a physical (not a simple) pendulum of any shape, the derivation follows the same steps. We'll refer to the moment of inertia for the physical pendulum as I_{PP} . The relevant distance of interest, as it was in the case of the simple pendulum, is the distance from the center of mass to the pivot point. We'll call this distance, \mathbf{d} . The restoring torque is still given by $\tau = k \Delta\theta$.

$$T = 2\pi (I_{PP}/k)^{1/2} = 2\pi (I_{PP} \Delta\theta / \tau)^{1/2} = 2\pi [(I_{PP})(\Delta\theta)/(d \cdot mg \sin \theta)]^{1/2}$$

Where we've again used the definition of torque to replace τ with $\mathbf{d} \cdot \mathbf{mg} \sin \theta$. Again, for small angles expressed in radians we will use the important trigonometric approximation $\sin \Delta\theta \sim \Delta\theta$. Therefore,

$$T = 2\pi [(I_{PP})(\Delta\theta)/(d \cdot mg \Delta\theta)]^{1/2} = 2\pi [(I_{PP})/(d \cdot mg)]^{1/2}$$

This is our second major result; a model for the period of a physical pendulum of any shape about any pivot point.

$$T = 2\pi \sqrt{\frac{I_{PP}}{mgd}}$$

Equation 2

Where I_{PP} is the moment of inertia about the pivot point, \mathbf{m} is the mass, and \mathbf{d} is distance from the pivot point to the Center of Mass (CM). Note that the period is undefined when the distance of the pivot from the center of mass is zero. If the pivot axis goes through the center of mass the pendulum will not oscillate. The actual axis of rotation must be parallel to but distinct from one of the axes through the center of mass.

The moment of inertia about any axis parallel to an axis through the CM is found using the *Parallel Axis Theorem*.

$$I_{PP} = I_{\text{axis parallel to axis through CM}} = I_{\text{axis through CM}} + \mathbf{m}d^2 = I_{CM} + \mathbf{m}d^2$$

This yields the general equation for the period of a physical pendulum about any axis parallel to a known axis through the Center of Mass:

$$T = 2\pi \sqrt{\frac{I_{CM} + \mathbf{m}d^2}{mgd}}$$

Equation 3

In this lab, the specific physical pendulum you'll be studying is most like a disk, so we will assume its moment of inertia is $I_{CM} = \frac{1}{2}\mathbf{m}R^2$. The general equation for the period can now be transformed, after a little algebra, into a specific equation for the period of your specific physical pendulum. i.e.:

$$T = 2\pi \sqrt{\frac{R^2 + 2d^2}{2gd}}$$

Equation 4

Notice that the mass of the pendulum cancelled out of the equation. We've used the symbols \mathbf{R} for the radius of the disk and \mathbf{d} for the distance from the pivot point to the **Center of Mass**. You will use *Equation 4* to model the period of the physical pendulum used in this lab. Note again that the period is undefined if \mathbf{d} is zero. The pendulum will not oscillate if the axis of rotation passes through the center of mass of the physical pendulum.

Part One: Simple Pendulum

Construct a simple pendulum. Use the photogate to measure the period at each of the following lengths. The weight at the end of the pendulum is made of washers hanging from a thread connected to a bent paper clip. Be sure to tape over the hole in the washers so that the photogate does not count the gap as a half cycle of the pendulum. The lab file *Pendulum Timer* is in the *New Labs* folder. Use the linear fit feature of LoggerPro and use the y-intercept (*period at time zero*) as the period of the pendulum.

L = Length (m) **T = Period (s)**

1.2	_____
1.0	_____
0.8	_____
0.7	_____
0.6	_____
0.5	_____
0.4	_____
0.3	_____
0.2	_____
0.1	_____
0.05	_____

Analysis: Simple Pendulum: Construct two graphs:

Graph 1: Period (*vertical axis*) vs **Length** (*horizontal axis*): The result will be a parabola opening to the right. Fit this graph with the variable power function (set n=0.5); i.e. $Y = A \cdot x^{0.5}$.

The coefficient $A = 2\pi/vg =$ _____; therefore, the value of $g =$ _____ m/s^2

Graph 2: Period squared (*vertical axis*) and **Length** (*horizontal axis*). The result will be a straight line through the origin (use the proportional function, $y = A \cdot x$). Then, the slope $A = 4\pi^2/g$.

Slope of line in Graph 2 = _____ (*from fit to $y = A \cdot x$*)

Experimental value of $g =$ _____ m/s^2

Standard value of $g =$ _____ m/s^2 : {Local g at school = _____ m/s^2 }

%Error (*Standard g is correct*) = _____ %: { %Error (*local g is correct*) = _____ % }

Part Two: Physical Pendulum

Construct a physical pendulum using a plastic disk with five pre-drilled holes at various distances from the center of the disk. Measure the period using each of the holes (*except the central hole*) as a pivot. Record these periods.

Imagine the pivot points are numbered from the outer edge toward the center. When making your measurements start with the pivot furthest from the center. The pivot closest to the center is the most difficult to measure. Friction will quickly slow the physical pendulum, so the earliest measurements should give the best results at each setting.

Radius of disk = **R** = _____ meters

Mass of disk = **m** = _____ kilograms (*we could model the period of this physical pendulum without knowing its mass, but you will be calculating values of I_{CM} and I_{PP} below and to do that you will need to know the mass. When you use Equation 4 to calculate the period, you will not use the mass.*)

Hole #	T = Period (sec)	d = Distance from the pivot to CM , i.e. the center of the disk (meters)
1.	_____	_____ (<i>This is the hole farthest from the center of the disk.</i>)
2.	_____	_____
3.	_____	_____
4.	_____	_____
5.	_____	_____ (<i>This is the hole nearest to the center of the disk.</i>)

Analysis: Physical Pendulum

Calculate the moment of inertia of the disk rotating around its center using $I_{CM} = \frac{1}{2} mR^2$

I_{CM} = _____

Now calculate I_{PP} at each new pivot point using the parallel axis theorem ($I_{PP} = I_{CM} + md^2$), and predict the period for each position. (*Assume the Predicted Period is the correct one when calculating %Error.*)

Pivot#	I_{PP}	mgd	Predicted Period, T_P	Measured Period, T_M	%Error (<i>Assume T_M is correct value</i>)
1.	_____	_____	_____	_____	_____
2.	_____	_____	_____	_____	_____
3.	_____	_____	_____	_____	_____
4.	_____	_____	_____	_____	_____
5.	_____	_____	_____	_____	_____

Questions for analysis: *You will need to consult a table of moments of inertia.*

1. What is the period of a meter stick pendulum pivoted at one end? *(You should first use the standard formula to calculate the moment of inertia for a meter stick pivoted at its center of mass and then apply the parallel axis theorem by using Equation 3.)*

Answer _____

2. What is the period of a meter stick pendulum pivoted at a hole drilled on the 90-cm mark? *(You will need to calculate the moment of inertia for a meter stick pivoted at its center of mass and then apply the parallel axis theorem by using Equation 3.)*

Answer _____

3. When the length of a simple pendulum is doubled the ... *(Circle the correct ending for this sentence)*
- a) period doubles.
 - b) period is divided by two.
 - c) period is multiplied by 1.414.
 - d) period is divided by 1.414.
 - e) period does not change.

4. If you compare periods of a simple pendulum in an elevator (A) accelerating upward to the same pendulum in the same elevator (B) moving upward at a constant speed, *(Circle the correct ending for this sentence)*
- a) period (A) = period (B).
 - b) period (A) > period (B).
 - c) period (A) < period (B).
 - d) period (B) will depend on how fast the elevator (B) is moving.

5. What is the length of a simple pendulum that gives a period of exactly one second?
(Use Equation 1, assume $g_{\text{local}} = 9.795 \text{ m/s}^2$)

Answer _____

6. What is the period of a physical pendulum made from a 1.0-kg, 50.0-cm-radius hoop pivoted on its edge?
(You will need to calculate the moment of inertia for a hoop pivoted at its center of mass and then apply the parallel axis theorem by using Equation 3.)

Answer _____

7. **Extra Credit** – This project can be time consuming so you should not begin unless you have enough time available to complete it. If you wish to pursue it, consult with your instructor before beginning. The extra credit available depends on the level of effort and the quality of the analysis produced. The due date can be later than the normal lab date, but only by arrangement in advance with the instructor.

Our physical pendulum is not a simple disk. It has a thicker ring around its outer rim. Some fraction (**F**) of the total mass lies in the disk. A complex shape like ours has a moment of inertia about its center of mass that is better represented by an equation of the type,

$$I_{CM} = \frac{1}{2} mr^2(\mathbf{F}) + mr^2(\mathbf{1-F})$$

(The disk) (The ring)

where (**F**) is the fraction of the mass in the disk and (**1-F**) is the fraction of the total mass in the ring.

First, show that this is equivalent to

$$I_{CM} = (\mathbf{1} - \frac{1}{2} \mathbf{F}) mr^2$$

Second, insert this equation for the moment of inertia about the center of mass into *Equation 3* and simplify to create the appropriate equation for the period of this complex object. The resultant equation replaces *Equation 4* in the revised analysis that you are about to undertake.

Third, use this revised equation to fit your data for the physical pendulum in Part 2. *Graphical Analysis* will allow you to enter almost any equation. Modify the equation and allow both **F** and **g** to serve as adjustable parameters. Compare the value of **F** obtained when you set **g** equal to the true local value with the value of **F** obtained when you let *Graphical Analysis* find the value of **g**. This will allow you to find the best combination that fits your data. Don't forget to calculate the %Error in **g** and comment on the reasonableness of the estimate of **F**. Can you think of any independent method of estimating **F** to confirm the range of estimates obtained from *Graphical Analysis*?

Fourth, write a report describing your procedures, methods, analysis, graphs and results. Include a section covering your conclusions about how well this worked or didn't work. This report must be submitted separately and will be graded independently of your regular lab report for this lab. This report must also include your derivation, calculations, and graphs.

In writing your report, use the full range of mathematical skills and your understanding of the relevant principles of Physics. State your procedures clearly and your conclusions honestly, but try to imbue your writing with the sense of discovery you've encountered in the process. Remember that you are writing the report after all the work has been done. It is not a mystery, but a report. It should have a clearly stated thesis including your conclusions and part of your job as a writer is to provide evidence to support that thesis and those conclusions. There are issues you will need to resolve. These should be addressed and incorporated into the statement, analysis and resolution of your thesis.

End your report with a recommendation of how you would extend this work to obtain even better estimates of **F** and **g**.