

Heating Water with Microwaves

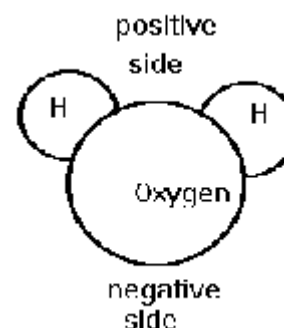
Name _____ Box _____

Due Date _____

Theory

How does a microwave oven warm (add energy to, increase the temperature of) water? It begins with the fact that water is a polar molecule, which means that it has positive and negative ends. The positive end is on the side with the hydrogen atoms.

If a water molecule is placed in an electric field, it will rotate until the positive end is pointing in the direction of the field. In other words, an electric field will exert a torque on the molecule. The microwave is a form of electromagnetic radiation. Like all electromagnetic radiation it has oscillating magnetic and electric fields at right angles to each other. The changing electric field of the microwave radiation causes the water molecules to rotate. As the electric field in the wave alternates very rapidly, the water molecules vibrate back and forth trying to follow the changing electric field. Microwaves are produced by a magnetron.



Basic Equations:

Energy of a microwave (or any other) photon

$$E = h f \quad (\text{where } h \text{ is Planck's Constant and } f \text{ is the frequency})$$

The **velocity** of any wave, including the waves in electromagnetic radiation is given by

$$v = f \lambda \quad (\text{where } f \text{ is the frequency and } \lambda \text{ is the wavelength})$$

Quantity of heat needed to change the absolute temperature (in K) of any other material with a known specific heat, c , is given by

$$Q = m c \Delta T \quad (\text{where } m \text{ is the mass and } \Delta T \text{ is the temperature change})$$

You will use a direct measurement of the energy absorbed by two water samples in the oven to estimate the efficiency with which the microwave energy (in the form of electromagnetic waves or photons) is absorbed by water in these particular samples in this particular oven.

Instructions: (See the **Technical Specifications** on page 8 for the base data on the oven.)

Record the frequency of the microwaves produced: $f =$ _____ Hz (or $1/s$ or s^{-1})

Record the power output of the oven in watts: $P_{\text{Oven}} =$ _____ W

Record the mass and initial temperature of the water in each trial. Do two trials.

Trial I

$M_1 = \text{Mass} =$ _____ g = _____ kg

Initial Temperature = _____ °C

Trial II

$M_2 = \text{Mass} =$ _____ g = _____ kg

Initial Temperature = _____ °C

Run the microwave oven with one water sample at a time.

Trial I

$\text{Time}_1 = \text{Heating time} =$ _____ sec

Trial II

$\text{Time}_2 = \text{Heating time} =$ _____ sec

Record the final temperature of the water in each sample and calculate the temperature changes.

Trial I

Final Temperature = _____ °C

$\Delta T_1 = \text{Temperature change} =$ _____ K

Trial II

Final Temperature = _____ °C

$\Delta T_2 = \text{Temperature change} =$ _____ K

Analysis - Part I:

A. Does the Microwave Oven Deliver what It Promises? Calculate the total energy delivered to the water as heat in each trial. (The specific heat of water is $c_{\text{Water}} = 4186 \text{ Joules}/(\text{kilogram}\cdot\text{Kelvin})$, **J/kg•K**.)

Trial I

$Q_1 =$ _____ J
 $= M_1 \cdot c_{\text{Water}} \cdot \Delta T_1$

Trial II

$Q_2 =$ _____ J
 $= M_2 \cdot c_{\text{Water}} \cdot \Delta T_2$

Now determine the power put into the water. This will be the total heat energy put into the water divided by the time (watts = joules / seconds).

$P_1 = Q_1 / \text{Time}_1 =$ _____ watts $P_2 = Q_2 / \text{Time}_2 =$ _____ watts

Compare the power output listed in the **Technical Specifications** with your measurement of the power actually absorbed by the water. Calculate the ratio of measured to listed power and record it as a percentage efficiency (you could consider this efficiency a measure of the oven's ability to deliver power to the water in your samples.).

%Efficiency = _____ % %Efficiency = _____ %

This might be called the power efficiency. It is only the simplest and most obvious form of analysis and by itself does not provide much insight into how the microwave radiation and the molecules of water interact with each other. The analyses in the following sections are a bit more theoretical and, it is hoped, a bit more insightful. You should pay careful attention to the story behind these calculations because you will be expected to explain it in a written narrative, as though trying to teach someone who was not there and did not understand the physics, how all this was accomplished and what it showed about how matter and electromagnetic radiation interact.

E. How many photons were produced for each molecule of water each second and how many photons were absorbed by each water molecule each second? The number of photons produced for each molecule each second = (# photons produced / # molecules) / (duration of the heating). The number of photons absorbed per water molecule per second = (# of photons absorbed / #molecules) / (duration of the heating).

Trial I

Trial II

Photons/molecule/sec = _____
 $N_{1P} / N_{1W} / \text{Time}_1 = \text{Photons produced/molec each second}$
 second

Photons/molecule/sec = _____
 $N_{2P} / N_{2W} / \text{Time}_2 = \text{Photons produced/molec each second}$

Photons/molecule/sec = _____
 $N_{1A} / N_{1W} / \text{Time}_1 = \text{Photons absorbed/molec each second}$
 second

Photons/molecule/sec = _____
 $N_{2A} / N_{2W} / \text{Time}_2 = \text{Photons absorbed/molec each second}$

From the total number of photons produced for each molecule each second and the number of photons actually absorbed by each molecule each second, calculate the photon absorption %Efficiency.

Trial I

Trial II

%Efficiency = _____ %

%Efficiency = _____ %

This absorption efficiency calculated here should be equal to the efficiency reported at the end of Part A.

Analysis - Part II - Calculations in Part II are all on a per molecule basis. The basic unit of time will be either one period or one quarter of the period of oscillation of the electromagnetic radiation. We are trying to understand how one molecule interacts with the electric field in an electromagnetic wave.

We'll try some fundamental calculations describing how a water molecule responds to the oscillating electric field of a microwave.

F. First, we need to calculate the average value of the oscillating electric field.

The amplitude of the electric field is given by

$$E_{MAX} = \sqrt{\frac{2P\mu_0 c}{A}}$$

where $P = P_{Oven}$ is the power output of the oven, c is the speed of light, A is the total surface area enclosing the cavity, and μ_0 is the permeability of free space (a constant found in the back of your textbook). Calculate the electric field amplitude using the lab data.

(Calculate the area of the interior of the oven as "Area = 2H•W+2W•D+2D•H")

Area = A = _____ m² Amplitude = E_{MAX} = _____ N/C

G. Next we need to find the torque this field exerts on each water molecule.

This electric field produces a torque on the water molecule according to the equation

$$\tau = E \cdot 6.3 \times 10^{-30} \cdot \sin \theta \quad (\text{Units: N}\cdot\text{m} = \text{N/C} \cdot \text{C} \cdot \text{m})$$

The number in this equation is called the *electric dipole moment* of water. This number has been measured experimentally. The optimum torque occurs when $E = E_{MAX}$ and θ equals 90°, i.e. when $\sin \theta$ equals 1.00. Thus,

$$\text{Maximum } |\text{Torque}| = |\tau_{MAX}| = \text{_____ N}\cdot\text{m}$$

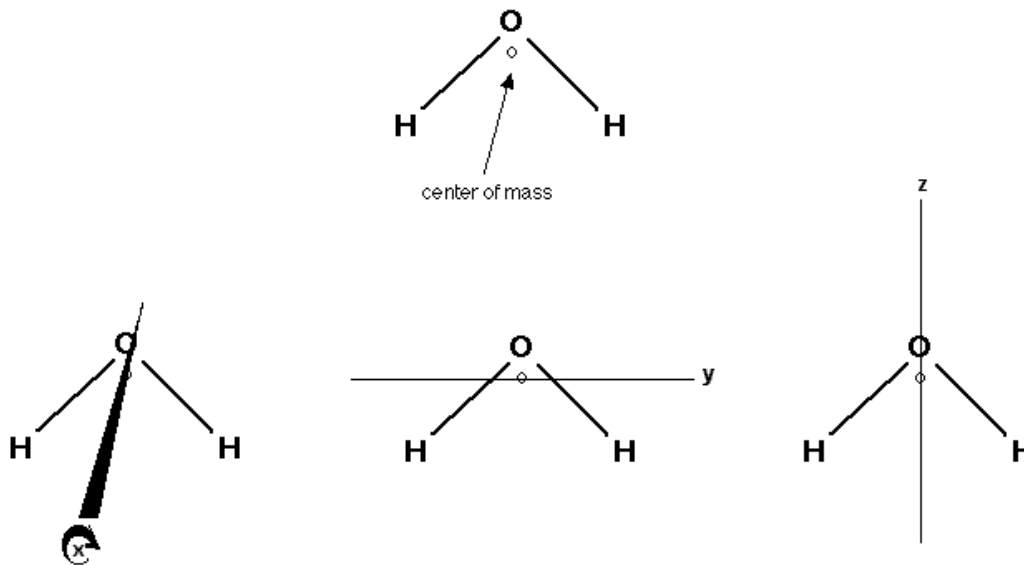
The average torque during a 180° rotation (one half-rotation) of the molecule, is

$$\text{Average } |\text{Torque}| = |\tau_{Average}| = \pi^{1/2} \cdot |\tau_{MAX}| / 2 = \text{_____ N}\cdot\text{m}$$

H. The Moment of Inertia for a single water molecule.

Next, we would like to determine if the acceleration produced by this torque is able to rotate the water fast enough for it to keep up with the changing electric field of the microwaves. We begin by estimating the moment of inertia of the water molecule. We need the moment of inertia about an axis that interchanges the positive and negative ends of the molecule. Rotations will accomplish that interchange about two principle axes. There are only three principle axes of rotation in a three dimensional molecule. All of the principle axes of rotation pass through the center of mass in the following diagram. In these three sections we check the possibilities.

Rotations about both the x-axis and the y-axis will interchange the positive and negative ends of the molecular dipole. The microwaves cannot cause the molecule to rotate around the z-axis. Make sure you understand why rotations about x and y interact with the microwaves, while rotation about z cannot interact with the microwaves.



To keep things simple, we will use the average of the two moments of inertia; I_{xx} and I_{yy} . The three principle moments of inertia of water are

$$\begin{aligned} I_{xx} &= 1.9274 \times 10^{-47} \text{ kg}\cdot\text{m}^2 \\ I_{yy} &= 2.9528 \times 10^{-47} \text{ kg}\cdot\text{m}^2 \\ I_{zz} &= 1.0254 \times 10^{-47} \text{ kg}\cdot\text{m}^2 \end{aligned}$$

Assuming that rotations about both the x and y axes are equally likely, we will use an average of these two moments of inertia as our average moment of inertia for the molecule.

Moment of Inertia = $(I_{xx} + I_{yy})/2 = I =$ _____ $\text{kg}\cdot\text{m}^2$

I. Now, use Newton's 2nd Law (recast in angular form) to calculate the average angular acceleration.

Use the rotational form of Newton's 2nd Law ($|\tau_{\text{Average}}| = I \alpha_{\text{Average}}$) to find the average angular acceleration of the water molecule from the known average torque on the water molecule.

$$\text{Average } \alpha = \alpha_{\text{Average}} = (|\tau_{\text{Average}}| / I = \underline{\hspace{2cm}} \text{ rad/s}^2$$

We assume this average acceleration applies to each half-rotation of a water molecule in a microwave field.

J. Find the angular distance a water molecule could rotate during each half-cycle.

$$\text{Frequency of the microwave} = f = \underline{\hspace{2cm}} \text{ s}^{-1} \text{ (from the instructions on page 2)}$$

$$\text{Wavelength of the microwave} = \lambda = \underline{\hspace{2cm}} \text{ m} = \underline{\hspace{2cm}} \text{ mm}$$

$$\text{Period of the microwave} = 1/f = T = \underline{\hspace{2cm}} \text{ s}$$

$$\text{Acceleration time} = \text{Period} / 2 = T/2 = \underline{\hspace{2cm}} \text{ s}$$

Now we are ready to calculate the angular distance the hydrogen nuclei could rotate during each half-cycle. For the water to keep up with the changing electric field of the microwave, it must be able to rotate at least one-half of a rotation during each T/2 seconds. As long as it can rotate at least that far that fast we can safely assume that is actually will rotate exactly one half-rotation per half-period.

$$\text{Angular distance} = \theta = \frac{1}{2} \alpha_{\text{Ave}} (T/2)^2 = \underline{\hspace{2cm}} \text{ radians; Rotations} = \theta / (2\pi) = \underline{\hspace{2cm}} \text{ rotations}$$

Be sure to understand the significance of this result. What it shows is that the molecule can rotate fast enough, in the absence of collisions, to keep up with the changing electric field. Furthermore, it shows that the molecule will pick up more than enough energy to keep up and therefore has excess rotational kinetic energy that it can transfer to the other molecules. What it does not answer is how much energy it will transfer. That is a separate question addressed in the next two sections.

K. Energy transfer from the microwave field to a water molecule.

There is Quantum Mechanical consideration that must be taken into account at this point. A molecule must absorb energy from the microwaves in increments of hf and must pass that energy along in increments of hf . The Work-Energy Theorem tells us that the change in internal energy of a system equals its change in kinetic energy. In other words;

$$\Delta U_{1/2} = \Delta K_{1/2} = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2 \quad \text{therefore} \quad \omega_f^2 - \omega_i^2 = 2 \Delta K_{1/2} / I$$

Assuming constant angular acceleration, we can use the rotational form of our equations of motion to calculate how far the molecule rotates as it absorbs one photon of energy. Thus,

$$\omega_f^2 = \omega_i^2 + 2 \alpha_{\text{MAX}} \Delta \theta \quad \text{or} \quad \omega_f^2 - \omega_i^2 = 2 \alpha_{\text{MAX}} \Delta \theta \quad \text{or} \quad 2 \Delta K_{1/2} / I = 2 \Delta \theta |\tau_{\text{MAX}}| / I$$

therefore, for one half-rotation ($= \pi$ radians)

$$\Delta U_{1/2} = \Delta K_{1/2} = |\tau_{\text{MAX}}| \Delta \theta = |\tau_{\text{MAX}}| \pi = \underline{\hspace{2cm}} \text{ J}$$

So, in absorbing one photon of energy, the molecule must complete

$$\Delta \theta_{\text{rotations}} = hf / (2 \cdot \Delta K_{1/2}) = \underline{\hspace{2cm}} \text{ rotations}$$

L. Finding the work done by the microwaves on a water molecule.

The classic textbook solution to this problem in the presence of a constant, external electric field is to calculate the change in potential energy of the water molecule that occurs when an external agent rotates the molecule against the direction of the electric field. In this solution one integrates force times distance, or as in this case, torque times angular distance. Assuming the molecule undergoes one half-rotation, or π radians, in the presence of an electric field of magnitude $E_{\text{Average}} = E_{\text{MAX}} / \sqrt{2}$, thereby averaging over all possible phase shifts, we find

$$\Delta U_{1/2} = 2 \cdot (E_{\text{Average}} \text{ N/C}) \cdot (6.3 \times 10^{-30} \text{ C}\cdot\text{m}) = \underline{\hspace{2cm}} \text{ J} = W_{1/2}$$

In absorbing one photon, however, the molecule makes more than one half-rotation. We know from the last section that it makes

$$\Delta \theta_{\text{rotations}} = \underline{\hspace{2cm}} \text{ rotations}$$

Therefore, the work done on the molecule by the electric field in one photon is the number of rotations times the work per rotation,

$$W_{\text{E-photon}} = (\Delta \theta_{\text{rotations}}) \cdot (2 \cdot W_{1/2}) = \underline{\hspace{2cm}} \text{ J}$$

Have we overlooked anything? Well, in fact, we have.

Recall that an electromagnetic wave contains both electric and magnetic fields. The magnetic field carries exactly the same amount of energy as the electric field. Not only that, but it is an oscillating magnetic field which, as we know, can cause charges to move. In the case of a dipolar molecule, like water, the charged ends will try to move in opposite directions with the result that the magnetic field will tend to make the molecule rotate. To take full account of the energy in the magnetic field, we should double the last result. Thus

$$W_{\text{photon}} = W_{\text{E-photon}} + W_{\text{B-photon}} = 2 W_{\text{E-photon}} = \underline{\hspace{2cm}} \text{ J}$$

Comparing this to the energy in one photon we find that the following fraction of the energy of the photon accomplishes work on the water molecule:

$$\% \text{Efficiency} = 100\% \cdot W_{\text{photon}} / hf = \underline{\hspace{2cm}} \%$$

The efficiency should be close to 100%. Deviation from 100% is probably due to our use of simplifying assumptions. These allow us to complete the calculations without the use of calculus but are also less accurate than a more rigorous mathematic treatment.

Still, given the constraints this seems to me to be in excellent agreement with expectations; namely that 100% of the energy in the microwaves is transmissible to the water. In our experiment, the efficiency was less than 100%. That is probably attributable to absorption by the walls of the oven when the amount of water is too small to capture all the photons. Our expectation is that with a larger sample of water, we will get those experimental efficiencies up near 100%.

M. Further experimentation

To verify our expectation that the %Efficiency should indeed be 100%, we might try one additional experiment.

Heat a larger volume of water in an attempt to capture as many of the photons as possible. If we can get the %Efficiency calculated in section A up close to 100%, that should be a sufficient verification that the %Efficiency in section L should also be 100%. A kilogram of water and 2 minutes heating time should provide a sufficient test. The idea is to capture all the photons in order to verify that all the energy in the photons (*in both the electric and magnetic fields*) is capable of being converted into heat.

Trial III

Mass = _____ gram

Initial Temperature = _____ °C

Heating time = _____ sec

Record the final temperature of the water and calculate the temperature change.

Final Temperature = _____ °C

Temperature change = _____ K

Calculate the total energy delivered to the water as heat in each trial. (The specific heat of water is 4186 Joules/(kilogram•Kelvin), **J/kg•K.**)

Q = _____ J

Now determine the power put into the water. This will be the total heat energy put into the water divided by the time (watts = joules / seconds).

Q / t = _____ watts

%Efficiency = _____ %

This little test should give us the maximum absorption of microwaves by the water. If this is 100% then our expectation that the answer in Section L should be 100% will be vindicated. If it is less than 100%, then it is also possible that leaks from the cavity or a phase shift of something other than 90° are responsible.

Questions:

1. If you heat 500 grams of water with this microwave, how long will you need to run the microwave to heat the water from 25°C to 100°C?
2. If you continue heating the water, how much longer will it take to boil all the water away?
3. How big (a size question this time) are the microwave photons generated by this oven:
 - a) What is the wavelength of the microwaves generated by the oven?
 - b) Does one of these waves fit inside the oven?
 - c) Could the internal cavity of the oven be considered a resonant cavity? Why or why not?
4. What is the overall efficiency of the oven? (Decide how this should be calculated and justify your method. Then calculate the overall efficiency.)
5. According to the information in the **Technical Specifications** what do you calculate is the operating voltage of this oven?
6. According to the information in the **Technical Specifications** what do you calculate is the overall equivalent electrical resistance of this oven?
7. Why is the power consumption of this oven, or any oven, greater than its power output?

Essay Assignment - Extra Credit

Write a lucid and carefully considered description of the microwave oven's ability to deliver power to enclosed foods. You should provide an introduction to the essay describing the scope of your article, discuss in outline form the type of measurements and calculations you performed, and explain how your conclusions follow from those measurements and calculations. Provide a quantitative basis for your conclusions. In a few clearly written paragraphs explain to someone who was not present at the time what you did, how you did it and what you learned from doing it. Illegible, incoherent, or disorganized essays receive no credit.

FAQ - Microwave Ovens

Q. Do Microwave ovens cook from the inside out?

No. Food is partially transparent to the electromagnetic waves in the microwave region of the spectrum, so the energy is able to shine through it. At the same time some of the energy is absorbed by the food. Usually most of the absorption occurs, and therefore most of the heat is produced, in an outer layer about an inch thick. So, for example, large pieces of meat will quickly cook to a depth of about an inch, while the inside portions are cooked more slowly by heat conduction; just like in a conventional oven. The effect can be dramatically different for different foods. The differences depend on the amount of water each food contains. If a food is mostly water, only the outside inch cooks at all. If a food contains both air and water (like bread, cake, etc.) then the energy penetrates all the way through and the food gets heated everywhere, even deep inside.

Q. If I put a fork in the Microwave oven, will it destroy the oven?

No. This is a myth, but it has some roots in reality. In order to safely use metals inside a microwave oven, the cook has to learn numerous complex and mysterious rules in order to avoid fires and undercooked food. For example, thin metal will heat up fast in the oven and may cause a fire. The famous staple in the paper popcorn bag comes to mind here. The staple heats up and sets fire to the bag. Another rule is that if a metal object in the oven is touched even lightly to another one, or touched to the metal wall of the oven, an electric arc might ignite at the point of contact. This can set fire to the other items in the oven or even to the oven itself. Another rule is to avoid putting metal objects with sharp points in the microwave oven. Sharp points on metal objects can initiate a coronal discharge, also sometimes known as Saint Elmo's Fire, which behaves the same as a flame and can set fire to the oven if allowed to continue.

Given the complexity of the rules for safe use, and this list is not complete, it is much easier and safer to totally ban the use of metals in microwave ovens. The alternative would be to send everyone to a safety school to learn these complex rules!

Q. Aren't these ovens tuned to a special frequency so they heat only the water?

No. The usual operating frequency of a microwave oven is nowhere near the resonant frequency of water, and the energy in the radiation will heat other substances. For example, drops of grease on a plastic microwave dish can be heated far hotter than 100°C, and this causes the mysterious scarring which frequently occurs on plastic utensils. Any molecule that is "polar" (has positive and negative ends) will be rotated to align with the electric field of the microwaves in the oven. The vibrating electric field rotates (vibrates) the water molecules and any other polar molecules within the food. Microwave ovens have difficulty melting ice, presumably because the molecules are bound together and cannot be rotated by the electric field of the microwaves.

If the oven was tuned specifically to the water resonance frequency the water in the thin outer surface of the food would absorb all the energy and only the outside surface would be heated. The thin outer surface of meat, for example, would become a blast of steam while the inside remained ice cold. Because the microwaves are not at the resonant frequency for water, the waves are able to penetrate an inch or so into the food before it is completely absorbed.

	School Microwave Oven	Home Microwave Oven
Technical Specifications		
Model Numbers	MW8779W	MT 0130SJB-0
Power Consumption:	11.0 Amps; 1250 W	13.0 Amps; 1,560 W
Output: *	800 W	1000 W
Outside Dimensions:	248mm(H)x456mm(W)x305mm(D)	___mm(H)x___mm(W)x___mm(D)
Oven Cavity Dimensions:	175mm(H)x292mm(W)x285mm(D)	___mm(H)x___mm(W)x___mm(D)
Operating Frequency:	2,450 MHz	2,450 MHz
Uncrated Weight:	Approx. 25 lbs.	Approx. 40 lbs.

* IEC 705-88 Test procedure
Specifications subject to change without notice

Instructions: (Keep a copy of your numerical results on these 3 pages and data look-up will be much easier.)

Record the frequency of the microwaves produced: $f =$ _____ Hz (or 1/s or s^{-1})

Record the power output of the oven in watts: $P_{\text{Oven}} =$ _____ W

Trial I

$M_1 = \text{Mass} =$ _____ g = _____ kg

Initial Temperature = _____ °C

Trial I

Time₁ = Heating time = _____ sec

Trial I

Final Temperature = _____ °C

$\Delta T_1 = \text{Temperature change} =$ _____ K

Trial II

$M_2 = \text{Mass} =$ _____ g = _____ kg

Initial Temperature = _____ °C

Trial II

Time₂ = Heating time = _____ sec

Trial II

Final Temperature = _____ °C

$\Delta T_2 = \text{Temperature change} =$ _____ K

Analysis - Part I:

A. Does the Microwave Oven Deliver what It Promises?

Trial I

$Q_1 =$ _____ J
 $= M_1 \cdot c_{\text{Water}} \cdot \Delta T_1$

$P_1 = Q_1 / \text{Time}_1 =$ _____ watts

%Efficiency = _____ %

Trial II

$Q_2 =$ _____ J
 $= M_2 \cdot c_{\text{Water}} \cdot \Delta T_2$

$P_2 = Q_2 / \text{Time}_2 =$ _____ watts

%Efficiency = _____ %

B. How "Big" (an energy question, not a size question) is a Microwave Photon?

Energy per photon = $hf =$ _____ J $P_p = \# \text{ of Photons / second} =$ _____
 $= P_{\text{Oven}} / hf$ Produced by the oven

C. Determine the number of microwave photons absorbed by the water during each trial.

Trial I

$N_{1P} = \# \text{ of photons produced} =$ _____
 $= P_p \cdot \text{Time}_1$ During the heating time

$N_{1A} = \# \text{ of photons absorbed} =$ _____
 $= (Q_1 / hf)$ During the heating time

%Efficiency = _____ %

Trial II

$N_{2P} = \# \text{ of photons produced} =$ _____
 $= P_p \cdot \text{Time}_2$ During the heating time

$N_{2A} = \# \text{ of photons absorbed} =$ _____
 $= (Q_2 / hf)$ During the heating time

%Efficiency = _____ %

D. How many photons were produced per water molecule and how many absorbed per water molecules?

$$\text{mass of water in grams} \times \frac{1 \text{ mole}}{18 \text{ grams}} \times \frac{6.02 \times 10^{23} \text{ molecules}}{\text{mole}} = \text{molecules of water}$$

Trial I

Trial II

$N_{1W} = \# \text{ of water molecules} =$ _____ $N_{2W} = \# \text{ of water molecules} =$ _____

Trial I

Trial II

Photons/molecule = _____ Photons/molecule = _____
 $N_{1P} / N_{1W} =$ Produced during the heating time $N_{2P} / N_{2W} =$ Produced during the heating time

Number of photons absorbed per molecule = $Q / \# \text{molecules} / (\text{energy per photon})$

Trial I

Trial II

Photons/molecule = _____ Photons/molecule = _____
 $N_{1A} / N_{1W} =$ Absorbed during the heating time $N_{2A} / N_{2W} =$ Absorbed during the heating time

%Efficiency = _____ % %Efficiency = _____ %

E. How many photons were produced for each molecule of water each second and how many photons were absorbed by each water molecule each second?

Trial I

Trial II

Photons/molecule/sec = _____ Photons/molecule/sec = _____
 $N_{1P} / N_{1W} / \text{Time}_1 =$ Photons produced/molec each second $N_{2P} / N_{2W} / \text{Time}_2 =$ Photons produced/molec each second

Photons/molecule/sec = _____ Photons/molecule/sec = _____
 $N_{1A} / N_{1W} / \text{Time}_1 =$ Photons absorbed/molec each second $N_{2A} / N_{2W} / \text{Time}_2 =$ Photons absorbed/molec each second

Trial I

Trial II

%Efficiency = _____ % %Efficiency = _____ %

Analysis - Part II -

F. First, we need to calculate the average value of the oscillating electric field.

(Calculate the area of the interior of the oven as “Area = 2H•W+2W•D+2D•H”)

Area = A = _____ m² Amplitude = $E_{MAX} =$ _____ N/C

G. Next we need to find the torque this field exerts on each water molecule.

Maximum |Torque| = $|\tau_{MAX}| =$ _____ N•m

Average |Torque| = $|\tau_{Average}| = \pi^{1/2} \cdot |\tau_{MAX}| / 2 =$ _____ N•m

H. The Moment of Inertia for a single water molecule.

$$\begin{aligned} I_{xx} &= 1.9274 \times 10^{-47} \text{ kg}\cdot\text{m}^2 \\ I_{yy} &= 2.9528 \times 10^{-47} \text{ kg}\cdot\text{m}^2 \\ I_{zz} &= 1.0254 \times 10^{-47} \text{ kg}\cdot\text{m}^2 \end{aligned}$$

Moment of Inertia = $(I_{xx} + I_{yy})/2 = I =$ _____ $\text{kg}\cdot\text{m}^2$

I. Now, use Newton's 2nd Law (recast in angular form) to calculate the average angular acceleration.

Average $\alpha = \alpha_{\text{Average}} = (|\tau_{\text{Average}}| / I =$ _____ rad/s^2

J. Find the angular distance a water molecule could rotate during each half-cycle.

Frequency of the microwave = $f =$ _____ s^{-1} (from the instructions on page 2)

Wavelength of the microwave = $\lambda =$ _____ $\text{m} =$ _____ mm

Period of the microwave = $1/f = T =$ _____ s

Acceleration time = Period / 2 = $T/2 =$ _____ s

Angular distance = $\theta = \frac{1}{2} \alpha_{\text{Ave}} (T/2)^2 =$ _____ radians; Rotations = $\theta / (2\pi) =$ _____ rotations

K. Energy transfer from the microwave field to a water molecule.

$$\Delta U_{1/2} = \Delta K_{1/2} = |\tau_{\text{MAX}}| \Delta\theta = |\tau_{\text{MAX}}| \pi =$$
 _____ J

$$\Delta\theta_{\text{rotations}} = hf / (2 \cdot \Delta K_{1/2}) =$$
 _____ rotations

L. Finding the work done by the microwaves on a water molecule.

$$E_{\text{Average}} = E_{\text{MAX}} / \sqrt{2}$$

$$\Delta U_{1/2} = 2 \cdot (E_{\text{Average}} \text{ N/C}) \cdot (6.3 \times 10^{-30} \text{ C}\cdot\text{m}) =$$
 _____ $\text{J} = W_{1/2}$

$$\Delta\theta_{\text{rotations}} =$$
 _____ rotations

$$W_{\text{E-photon}} = (\Delta\theta_{\text{rotations}}) \cdot (2 \cdot W_{1/2}) =$$
 _____ J

$$W_{\text{photon}} = W_{\text{E-photon}} + W_{\text{B-photon}} = 2 W_{\text{E-photon}} =$$
 _____ J

$$\% \text{Efficiency} = 100\% \cdot W_{\text{photon}} / hf =$$
 _____ $\%$