

Diffraction Gratings

Name _____ Box _____

Lab Partners: _____ Due Date: _____

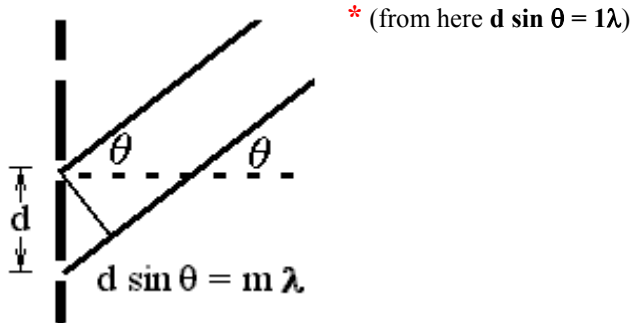
Theory:

Light from a single source can pass through a barrier divided by two or more openings. The light emerging from the openings spreads out in waves that interfere with each other. Thus an interference pattern forms. It might look something like this: *******, or *** * ***, or *** * * * ***.

In a transmission diffraction grating there are many tiny parallel grooves or lines scratched into a transparent film. The light scattered by even a few grooves interferes to form a pattern of closely spaced bright and dim patches called fringes. As we add more and more closely spaced grooves the bright fringes get farther and farther apart. Depending on the spacing of the grooves and the wavelength of the light, each bright fringe appears at a specific, predictable angle; each bright fringe appears at a point where the distances to adjacent grooves differ by an integer number of wavelengths. Constructive interference only occurs when adjacent grooves are one, two, three, etc., wavelengths closer or farther from the bright fringe than their nearest neighbors. In general, the closer the grooves are to each other the larger the diffraction angle, θ , becomes. ***** (from here $d \sin \theta = 2\lambda$)

Light wave of wavelength λ approaching grating grooves

→ | → | → |
→ | → | → |
→ | → | → |
→ | → | → |
 λ | λ | λ |



The points of constructive interference are called **maxima** because the constructive interference reaches its maximum intensity at these angles. The angles between the first two **maxima** and the incoming light beam are given by:

$$d \sin \theta = \lambda \quad (\text{for the first maximum})$$

$$d \sin \theta = 2\lambda \quad (\text{for the second maximum})$$

Where d is the distance between adjacent grooves in the grating (λ , and d must be in the same units).

The number of **maxima** depends on d , and λ ; $m_{\text{MAX}} = \text{largest integer less than } d / \lambda$.

Reflection gratings produce a similar result but have a mirror behind a surface scratched with very closely spaced grooves. A CD acts as a reflection grating. It has a single line of data written in a tightly wound spiral. Over a small area on its surface, the grooves on a CD act like the lines on a real reflection grating.

Procedure:

Transmission Grating

1. Draw a line across the middle of a long sheet of paper. Aim a laser along the line. Place a known transmission diffraction grating in front of the laser at the near edge of the paper. Mark the location of the first and second maximum on each side of the incoming laser beam.
2. Measure the angle on the paper between the laser beam and each maximum. Record these in the table.

Transmission Grating – Data Table

The first transmission grating is labeled: _____ grooves per mm

First maximum: $\theta_{\text{left}} =$ _____ $\theta_{\text{right}} =$ _____ $\theta_{\text{average}} =$ _____

Second maximum: $\theta_{\text{left}} =$ _____ $\theta_{\text{right}} =$ _____ $\theta_{\text{average}} =$ _____

The second transmission grating is not labeled. The line spacing is unknown. Usually, only one maximum is visible on each side of the transmitted beam, but **record the second one if you find it.**

First maximum: $\theta_{\text{left}} =$ _____ $\theta_{\text{right}} =$ _____ $\theta_{\text{average}} =$ _____

Second maximum: $\theta_{\text{left}} =$ _____ $\theta_{\text{right}} =$ _____ $\theta_{\text{average}} =$ _____

Reflection Grating

3. Aim a laser beam along a line drawn across the center of the sheet. Place a CD in its case so that the outer edge of the CD (*where the grooves are straightest*) is at the far end of the line. The laser beam should hit the CD at a point directly to the right of the central hole (*so the grooves are vertical*). Adjust the CD so that the reflection of the beam goes back to the laser opening. The CD should now be aimed directly at the light source.
4. Locate the first and second interference maxima. Mark these points on the paper and measure the angle the reflected beam makes with the incoming beam. If you are lucky enough to get a third or even a fourth maximum on either or both sides, measure those too.

Reflection Grating – Data Table

First maximum: $\theta_{\text{left}} =$ _____ $\theta_{\text{right}} =$ _____ $\theta_{\text{average}} =$ _____

Second maximum: $\theta_{\text{left}} =$ _____ $\theta_{\text{right}} =$ _____ $\theta_{\text{average}} =$ _____

Third maximum: $\theta_{\text{left}} =$ _____ $\theta_{\text{right}} =$ _____ $\theta_{\text{average}} =$ _____

Fourth maximum: $\theta_{\text{left}} =$ _____ $\theta_{\text{right}} =$ _____ $\theta_{\text{average}} =$ _____

Inner radius of the recorded part of the CD = $r_i =$ _____ mm

Outer radius of the recorded part of the CD = $r_o =$ _____ mm

Radial width of the recorded band on the CD; $R_w = r_o - r_i =$ _____ mm

Analysis

About the HeNe Laser

A certain mixture of the Noble gases helium and neon is excited in a closed glass container using the same technique used in neon lights. A pair of electrodes is used to send a stream of electrons through the thin, low-pressure gas. Collisions with the electron stream excite the electrons in the atoms. When those electrons return to lower energy levels, light is emitted. That light is collected by the laser mirrors and fed back through the excited gas stimulating more light to be emitted on each pass through the gas. One of the mirrors allows a tiny percentage of the light to leak out. The wavelength of the emitted light is determined by the energy levels of the gas atoms.

$$\lambda \text{ (HeNe laser)} = 632.8 \text{ nm} = \underline{\hspace{2cm}} \text{ mm} = \underline{\hspace{2cm}} \text{ m}$$

Transmission Grating

The Labeled Grating:

What is the value of $1/d$ for the first maximum of this transmission grating? lines per mm.

What is the value of $1/d$ for the second maximum of this transmission grating? lines per mm.

The grating label says it has lines per mm.

Your average result indicates lines per mm.

Percent Error = % (*Assuming the label is correct.*)

The Unknown, Unlabeled Grating:

What is the value of $1/d$ for the first maximum of this transmission grating? lines per mm.

What is the value of $1/d$ for the second maximum of this transmission grating? lines per mm.

Your average result indicates lines per mm.

Assuming the percent error is about the same as for the marked grating, is there a nice round number within or near the expected error limit that would make a reasonable guess for the true line density?

What line density would you guess? lines per mm.

Assuming your guess is the correct value, find your percent error.

Percent Error = %

Reflection Grating

What is the value of $1/d$ for the first maximum of the CD? _____ lines per mm.

What is the value of $1/d$ for the second maximum of the CD? _____ lines per mm.

Average value of $1/d =$ _____ lines per mm

Use the width of the band on the CD to determine the total number of loops in the spiral on the CD.

R_w (from page 2) = _____ mm; and, $N = R_w \cdot 1/d = R_w / d =$ _____ loops

The total length of the line on the CD is the total length of the spiral the line makes.

This will be the number of loops times the average circumference of all the spiral line segments.

$$\text{Spiral length} = N 2\pi(r_i + d N / 2) = N 2\pi(r_i + R_w / 2)$$

Where r_i is the inner radius at the start of the spiral, N is the number of loops of the spiral, and d is the distance between lines of the spiral.

Spiral length = _____ mm = _____ meters = _____ km

A typical music CD plays for 75 minutes. Now find the average velocity of the line of data moving past the laser reading head.

Velocity = $v =$ _____ mm/s = _____ m/s = _____ km/hr

The CD player mechanism keeps this velocity constant as the CD reader mechanism moves to different radial distances from the center of the CD. In order to do this, the angular velocity of the disc must be varied to keep the velocity of the CD moving past the reader constant. Use $v = r\omega$ to find the correct angular velocity for the disk :

a) when the CD reader is at the inner edge of the disk. $\omega_i = v / r_i =$ _____ rad/sec

b) when the laser is reading at the outer edge of the disk $\omega_o = v / r_o =$ _____ rad/sec

Convert these two angular speeds (both the maximum and minimum angular speeds) to revolutions per second and revolutions per minute

$RPS_i =$ _____ rev/sec; $RPM_i =$ _____ rev/min

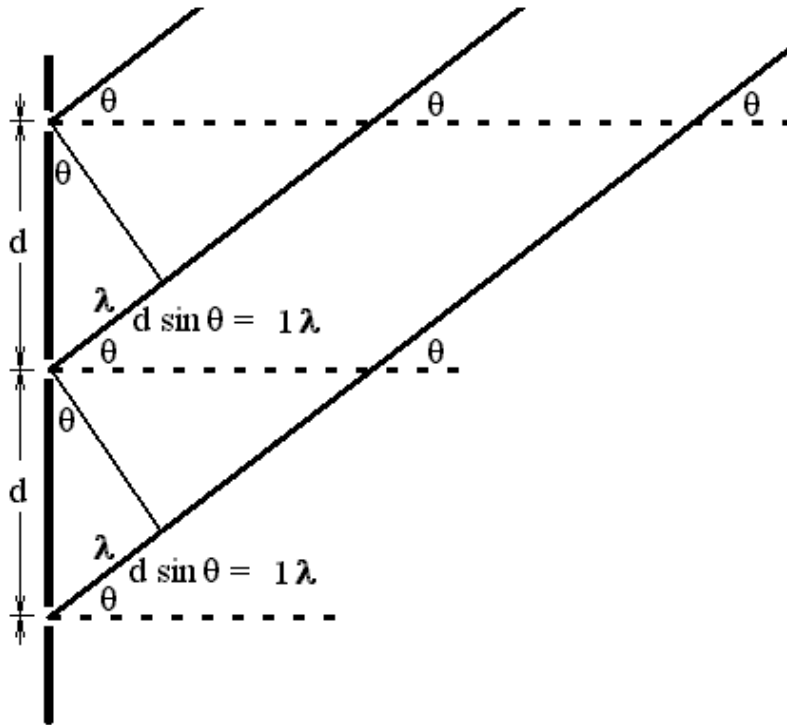
$RPS_o =$ _____ rev/sec; $RPM_o =$ _____ rev/min

Questions:

1. Write a sentence that explains why white light hitting diffraction gratings will produce a spectrum.
2. If you look through a transmission grating at a white light, will the red side or the violet side of the spectrum be closer to the central undiffracted beam? Explain.
3. (**Don't try this.**) If one looks at the sun through a bird feather, a diffraction pattern appears. Why?
4. Diffraction gratings come with a wide range of line spacings. Let's examine the difference this makes.
 - A. Calculate the diffraction angle of every possible maximum for these three gratings with different line spacings: (assume that the wavelength of the light is 683.2 nm.)
 - a) 300 lines per mm,
 $\theta_1 = \underline{\hspace{2cm}} = \theta_2 = \underline{\hspace{2cm}} = \theta_3 = \underline{\hspace{2cm}} = \theta_4 = \underline{\hspace{2cm}}$
 - b) 600 lines per mm
 $\theta_1 = \underline{\hspace{2cm}} = \theta_2 = \underline{\hspace{2cm}} = \theta_3 = \underline{\hspace{2cm}} = \theta_4 = \underline{\hspace{2cm}}$
 - c) 900 lines per mm
 $\theta_1 = \underline{\hspace{2cm}} = \theta_2 = \underline{\hspace{2cm}} = \theta_3 = \underline{\hspace{2cm}} = \theta_4 = \underline{\hspace{2cm}}$
 - B. Find the maximum number of lines per mm that will allow at least one maximum to form when using the helium-neon laser wavelength. (The trick is to remember that $\sin \theta$ cannot be larger than 1.00000.)

Detailed Drawing of the Creation of an Interference Pattern by the Scratches on a Diffraction Grating.

First Order Diffraction Fringe:

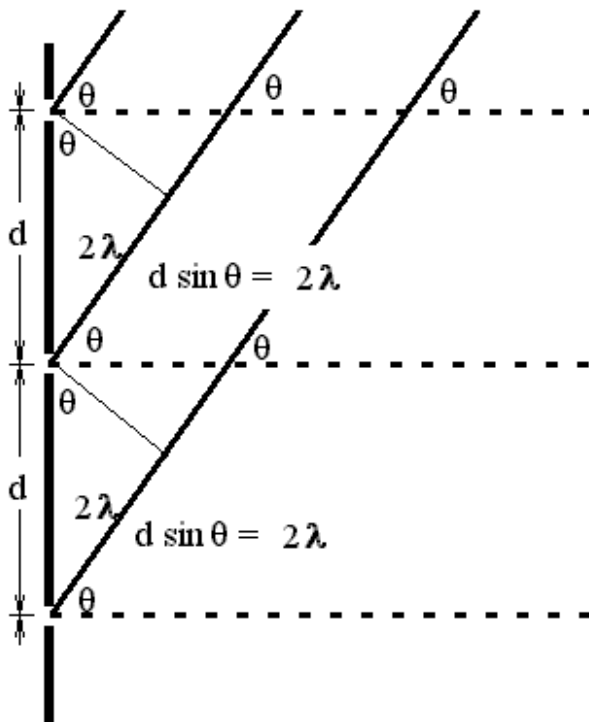


For the first bright fringe, the scratches on the grating are all one wavelength nearer or farther from the fringe than their nearest neighbors.

The small triangles have a hypotenuse of d and a length difference equal to $1\lambda = d \sin \theta$.

(Don't forget that the rays travel a very long distance to reach the fringe. This diagram is very tiny compared to the width of the beam and compared to the distance to the fringe.)

Second Order Diffraction Fringe:



For the second bright fringe, the scratches on the grating are all two wavelengths nearer or farther from the fringe than the nearest neighbors. The angle is larger than for the first order fringe.

The small triangles still have a hypotenuse of d , but the length difference now equals $2\lambda = d \sin \theta$.

(Don't forget that the rays travel a very long distance to reach the fringe. This diagram is tiny compared to the width of the beam and compared to the distance to the fringe.)

The Spiral Puzzle

At a lumber mill, some logs are processed through a machine that essentially pares away the wood in a continuous spiral sheet, almost as though the log had been a roll of wrapping paper.

Acertain amount of wood is wasted during the cutting process, especially if the sheet tears and the cutting process has to be restarted. Another source of waste occurs because the process must be terminated before the log is completely "unwrapped", leaving a scraggly pole that is tossed into the scrap bin.

An efficiency-minded manager believes that the amount of wood wasted in this process is as much as 25 percent. He is interested in confirming his guess by having you give a reasonable estimate of what the process would look like if it worked perfectly.

So suppose we begin with a log that is a perfect cylinder, 10 feet long, and 4 feet in diameter, and that we wish to spiral cut it into a continuous sheet that is $\frac{1}{4}$ inch thick. We may assume that the sheet can be flattened, although it actually has some curvature that increases dramatically as the cut approaches the center of the log. Let us also assume that, in the messy rough and tumble of a lumber mill, it is typical to get several $\frac{1}{4}$ inch thick sheets of wood that are equivalent to one sheet that is about 120 feet in length from this log.

The first question is, can you compute the exact length of the sheet of wood you could cut from the log in a perfect mathematical process? It will be satisfactory to compute the length of a spiral line that represents the position of the cutting blade. This requires use of the integral calculus.

Unbelievably, the more formulas you write, the less some people believe you. So, the second question is this; can you think of a simpler method of estimating the length of the sheet of wood that appeals to common sense and involves nothing more than arithmetic and geometry?

Solutions to "The Spiral Puzzle"

If we think about the spiral we are describing as proceeding from the center outwards, then every time it completes a revolution (2π radians), the radius of the spiral has increased by $1/4$ inch. Therefore, the appropriate spiral formula relating r , the distance from the center, to θ , the angle of revolution, is

$$\text{Since } L = r * \theta; dL = r d\theta; \text{ but } r = 1/4 * \theta / (2\pi); \text{ therefore } dL = \theta / (8\pi) d\theta$$

Next, we observe that the number of revolutions will be 48 per radial foot. Since each revolution advances us $1/4$ inch from the center, and we have to advance a total of 2 feet to reach the outer surface, the number of revolutions is 96. Hence, the length of the spiral is:

$$\begin{aligned} \text{Length} &= \int_0^{96 * 2 * \pi} \theta / (8 * \pi) d\theta = 1 / (8 * \pi) * \int_0^{192 * \pi} \theta d\theta \\ &= 1 / (8 * \pi) * (1/2 * \theta^2 \Big|_0^{192 * \pi}) \end{aligned}$$

which gives us

$$= 1 / (8 * \pi) * (1/2 * (192 * \pi)^2 - 0)$$

$$= 192^2 * \pi / 16 \text{ inches}$$

or

$$\text{Length} = 2304 * \pi \text{ inches} = 192 * \pi \text{ feet} = 603.1857... \text{ feet approximately.}$$

Here, we are assuming that the length of the spiral is a good estimate of the length of the sheet. Of course, a spiral is linear like a string, while a sheet has thickness, and may be inherently curved, with the outer side being longer than the inner side.

Now if the manager doesn't believe all your high-falutin' math, you can simply lean back in your chair and ignore him, secure in the knowledge that you are right; or you can repeat everything you said but **LOUDER**; or you could try a different approach to the problem that allows your audience to follow you more easily.

A reasonable way of estimating the behavior of the problem is to suppose that, if we have a perfect cut, then all the wood in the log goes into the wooden sheet, and the curvature of the wooden sheet can be ignored. The resulting sheet can be viewed as a 10-foot wide, $1/4$ -inch high and "very long" box. To determine its maximum theoretical length we ignore the cutting process, and pretend that the whole log was simply "melted" and poured back into a mold.

Now, we simply note that the volume of the log must equal the volume of the mold, so, remembering that $1/4$ inch = $1/48$ foot, we have

$$(\pi * r^2 * W) \text{ ft}^3 = \text{Volume of Cylinder} = \text{Volume of Box} = L * W * H \text{ ft}^3$$

$$\pi * r^2 = L * H$$

$$\pi * 2^2 = \text{Length} * 1/48$$

which tells us that

$$\text{Length} = 48 * \pi * 4 = 603 \text{ feet (approximately)}$$

Using the formula on page 4 of the lab handout gives the same result. Try it, assuming the inside radius is zero with an outer radius of 2 feet. Go ahead, give it a whirl.