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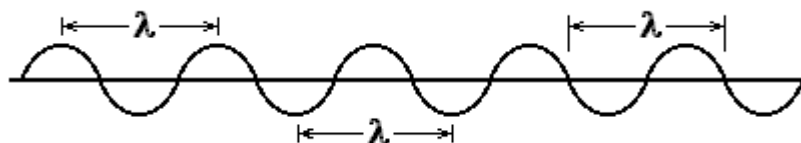
# WAVELENGTH and FREQUENCY CALCULATIONS

**Purpose:** To gain familiarity with the units used for reporting wavelength and frequency; To review relationships of wave-velocity and wave-energy to frequency and wavelength; To introduce the related quantity known as wave number; To practice unit conversions on measures of length, frequency, and wave number.

## Wavelength Units

Wavelength ( $\lambda = \text{lambda}$ ) is a length or a distance. It is the length of one wave or, equivalently, it is

the distance between two adjacent points of identical phase on a wave; two adjacent peaks, for one example, or two adjacent minima, for another. Because electromagnetic waves come in a wide range of lengths, wavelength is also reported in a wide range of units. Practically every unit of length that you can name, and some you cannot, is or was used as the standard of length for some part of the electromagnetic spectrum.

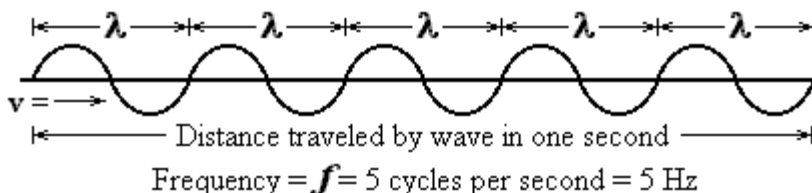


It would be nice to say that we've standardized all wavelengths to the meter, which technically is true, but in fact people are creatures of habit and convenience, so many units of length are still being used today. That means you need to get used to seeing and dealing with this diversity of convenience units.

## Frequency Units

The frequency ( $f$ ) measures the number of peaks (or valleys) in the wave train that pass a point each second. The wave must move one wavelength for each new peak to

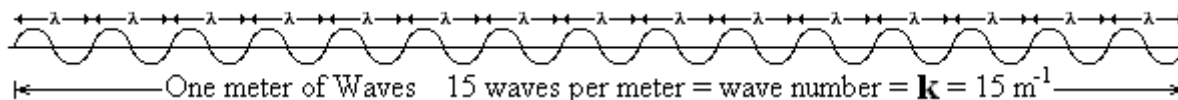
reach the point, so it would appear that the natural unit for frequency is wavelengths per second. Most people find it more comfortable to use the generic cycles per second. More precisely, the cycle per second only has true units of  $1/s = s^{-1}$ . The modern unit of frequency is named the hertz (Hz), after the German spectroscopist Heinrich Rudolf Hertz. (1857-1894), who was the first to produce radio waves artificially.



Do not confuse the frequency,  $f$ , with the angular frequency,  $\omega$ . Angular frequency is measured in radians per second. It is not measured in Hz. The angular frequency in radians per second has a numerical value that is exactly  $2\pi$  times the frequency in hertz for the same wave.

## Wave Number Units

The wave number ( $k = 1/\lambda$ ) of a wave gives the number of waves in a specified length; say waves (cycles) per meter or waves (cycles) per centimeter. The units of wave number are most commonly given in  $m^{-1}$  or  $cm^{-1}$ ; think of these as cycles per meter and cycles per centimeter. If you already have the wavelength in the appropriate unit, then the wave number is simply the inverse of the wavelength. The British Association for the Advancement of Science adopted the term "wave number" in 1872 as the preferred replacement for "inverse wavelength." Originally,  $mm^{-1}$  was the standard unit, but today  $cm^{-1}$  and  $m^{-1}$  are customarily used.



### Practice Wavelength Unit Conversions

A certain laser puts out green light with a wavelength of **0.000 000 510 0** meters. Convert this length to the following units and write the wavelength of this green laser light in proper scientific notation for each unit.

$$\lambda (0.000\ 000\ 510\ 0 \text{ meters}) = \underline{\hspace{2cm}} \text{ m} = \underline{\hspace{2cm}} \text{ cm}$$

$$\lambda (0.000\ 000\ 510\ 0 \text{ meters}) = \underline{\hspace{2cm}} \text{ mm} = \underline{\hspace{2cm}} \mu\text{m}$$

$$\lambda (0.000\ 000\ 510\ 0 \text{ meters}) = \underline{\hspace{2cm}} \text{ nm} = \underline{\hspace{2cm}} \text{ pm}$$

### Practice Frequency Unit Calculations

Find the frequency (in Hz) of the light emitted from three different lasers using the information given below.

**Laser #1:** In one microsecond light of this frequency passes through 473 755 464.602 oscillations.

$$\text{Its frequency is } \underline{\hspace{2cm}} \text{ Hz}$$

**Laser #2:** In one millisecond light of this frequency passes through 629 551 570 769 oscillations.

$$\text{Its frequency is } \underline{\hspace{2cm}} \text{ Hz}$$

**Laser #3:** In one nanosecond light of this frequency passes through 587 828.349 02 oscillations.

$$\text{Its frequency is } \underline{\hspace{2cm}} \text{ Hz}$$

### Practice Wave Number Unit Calculations

Use the wavelength of the green laser light to calculate the wave number of the green light in the following units. Write the resultant wave numbers in proper scientific notation. Then try a couple more examples using light from different lasers operating at different wavelengths.

$$k (\text{of } \lambda = 0.000\ 000\ 510\ 0 \text{ m light}) = \underline{\hspace{2cm}} \text{ m}^{-1} = \underline{\hspace{2cm}} \text{ cm}^{-1}$$

$$k (\text{of } \lambda = 0.000\ 063\ 28 \text{ cm light}) = \underline{\hspace{2cm}} \text{ m}^{-1} = \underline{\hspace{2cm}} \text{ cm}^{-1}$$

$$k (\text{of } \lambda = 0.000\ 476\ 2 \text{ mm light}) = \underline{\hspace{2cm}} \text{ m}^{-1} = \underline{\hspace{2cm}} \text{ cm}^{-1}$$

Notes: Wave number is less commonly used than either frequency or wavelength to describe a wave. Nevertheless, if you go into almost any technical field from medicine, to oceanography, to astronomy, to architecture, not to mention chemistry, physics and all types of engineering, you will run into this term again.

## The Speed of Light in a Vacuum

The speed of light in a vacuum is a universal constant. All forms of electromagnetic radiation (*light in the most general sense*) travel through empty space with the same velocity. This speed of light ( $c$ ) is known to very high precision and is equal to 299 792 458 m/s, approximately. Two equations involving the speed of light that you must know are these:

$$c = f\lambda \quad \text{and} \quad c = f/k$$

In order to calculate the speed of light in meters per second, you must use a frequency in Hz ( $s^{-1}$ ), and a wavelength in meters (m) or a wave number in one over meters ( $m^{-1}$ ).

Use these equations and the frequencies of the three lasers named on the previous page, to calculate the wavelength and wave number of the light from each laser.

**Laser #1:** Its frequency is \_\_\_\_\_ Hz

Therefore, its wavelength =  $\lambda$  = \_\_\_\_\_ m

And its wave number =  $k$  = \_\_\_\_\_  $m^{-1}$

**Laser #2:** Its frequency is \_\_\_\_\_ Hz

Therefore, its wavelength =  $\lambda$  = \_\_\_\_\_ m

And its wave number =  $k$  = \_\_\_\_\_  $m^{-1}$

**Laser #3:** Its frequency is \_\_\_\_\_ Hz

Therefore, its wavelength =  $\lambda$  = \_\_\_\_\_ m

And its wave number =  $k$  = \_\_\_\_\_  $m^{-1}$

Notes: The speed of light when it moves through matter (*air, glass, water, etc.*) moves more slowly than it does through a vacuum. The reason it moves more slowly is not that the frequency changes, but that the wavelength gets shorter. Think of it this way: wavelength is the step size and frequency tells how many steps it takes each second. Light slows down not because it is taking few steps each second, but because it is taking shorter steps.

This is important, as you will see in the next section, because the energy carried by light is determined by the frequency of its oscillations. When light passes through matter, its speed may change, but the energy it carries does not change.

Energy and frequency of light are interrelated. Since the energy of the light does not change while it passes straight through matter, neither does its color change. The color of the light can only be changed if it interacts directly with one of the atoms in the material. Then all bets are off. However, as long as the light travels in a straight line through matter, its energy and frequency remain constant.

## Energy Carried by Electromagnetic Radiation

All waves carry energy and light is no exception. It is made up of synchronized oscillating electric and magnetic fields, each of which are known to store energy. When any material object loses energy, one of the options is to form little energy packets, which we call photons of light. These packets of energy then move the energy away from the source at the speed of light.

When you study it in school, light may seem to be fantastically complicated. Experience shows, however, that making photons of light to carry energy is the simplest and most natural thing in the whole universe. You, yourself, are a continuous source of such photons. Some of the photons coming from you were created somewhere else and are merely reflected off you. This allows you to be seen. You also emit a continuous supply of your own photons that we cannot see, but are just as real as the visible ones. These are called infrared photons. All warm objects, including you, emit infrared photons. Infrared photons can be seen with special detectors.

The only difference between visible light and infrared light is the frequency and therefore the energy carried by the light. Most of the visible light in our world comes from the Sun. It is a very hot and therefore much more energetic source of photons than you or I. Visible photons carry more energy than infrared photons. We are warm but not nearly as warm as the Sun, so we emit lower-energy infrared photons.

The energy of a photon is proportional to its frequency. The proportionality constant is known as Planck's Constant,  $h$ . Thus,

$$E = hf$$

The value of  $h$  is  $6.626\,069\,3 \times 10^{-34}$  J·s. Clearly, even at high frequencies, one photon cannot carry much energy; or so it would seem. Remember that one photon can only interact with one particle or one atom at a time. These particles are all very small. The energy of a visible photon is always enormous to the smallest particle: the electron. One visible photon carries enough energy to move an electron around rather violently within the atom. Slightly more energetic photons, called ultraviolet photons, can even eject an electron from its atom.

Because frequency, wavelength and energy are all interrelated, we can write the energy equation in several other ways.

$$E = hf = hc/\lambda = hc k$$

To calculate the energy of the photon in joules, you must use speed of light in m/s, frequency in Hz, wavelength in m, and wave number in  $m^{-1}$ .

### Practice Energy Calculations

You know the frequency, wavelength and wave number for the light from three lasers. The speed of light and Planck's Constant have both been given. Use all three versions of the energy equation to calculate the energy of a photon from each laser.

#### Laser#1

$$E(f) = \underline{\hspace{2cm}} \text{ J}; E(\lambda) = \underline{\hspace{2cm}} \text{ J}; E(k) = \underline{\hspace{2cm}} \text{ J};$$

#### Laser#2

$$E(f) = \underline{\hspace{2cm}} \text{ J}; E(\lambda) = \underline{\hspace{2cm}} \text{ J}; E(k) = \underline{\hspace{2cm}} \text{ J};$$

#### Laser#3

$$E(f) = \underline{\hspace{2cm}} \text{ J}; E(\lambda) = \underline{\hspace{2cm}} \text{ J}; E(k) = \underline{\hspace{2cm}} \text{ J};$$

## The ångström (Å)

The ångström is a unit of length named after the Swedish astronomer **Anders Jonas Ångström** (1814 – 1874). Formally, it equals  $1 \times 10^{-8} \text{ cm} = 1 \times 10^{-10} \text{ m}$ . Until the most recent decade or so, this was the preferred unit for reporting wavelengths in the visible, near infrared and ultraviolet regions of the electromagnetic spectrum. It is also convenient for reporting atomic and molecular dimensions. Its use is discouraged by the **Bureau International des Poids et Mesures** (International Bureau of Weights and Measures, or **BIPM**), one of the three organizations established to maintain the SI systems of units. It is still listed as one of the acceptable non-SI units, which may be used in scientific publications if it is defined in that publication. It is still popular in some scientific circles and among some researchers.

With our increasing reliance on the SI system of units, the ångström has fallen out of popular scientific use. It has largely been replaced by the SI unit, the nanometer, which is ten times larger:  $1 \text{ nm} = 10 \text{ Å}$ , or the picometer, which is one hundred times smaller:  $1 \text{ Å} = 100 \text{ pm}$ . Nevertheless, even some recent books on science and technology and almost all such written more than a decade or two ago, will use the ångström unit somewhere in the text. This alone makes it worth your while to become familiar with this important historical unit of length.