The Spiral Puzzle

At a lumber mill, some logs are processed through a machine that essentially pares away the wood in a continuous spiral sheet, almost as though the log had been a roll of wrapping paper.

A certain amount of wood is wasted during the cutting process, especially if the sheet tears and the cutting process has to be restarted. Another source of waste occurs because the process must be terminated before the log is completely "unwrapped", leaving a scraggly pole that is tossed into the scrap bin.

An efficiency-minded manager believes that the amount of wood wasted in this process is as much as 25 percent. He is interested in confirming his guess by having you give a reasonable estimate of what the process would look like if it worked perfectly.

So suppose we begin with a log that is a perfect cylinder, 10 feet long, and 4 feet in diameter, and that we wish to spiral cut it into a continuous sheet that is 1/4 inch thick. We may assume that the sheet can be flattened, although it actually has some curvature that increases dramatically as the cut approaches the center of the log. Let us also assume that, in the messy rough and tumble of a lumber mill, it is typical to get several 1/4 inch thick sheets of wood that are equivalent to one sheet that is about 120 feet in length from this log.

The first question is, can you compute the exact length of the sheet of wood you could cut from the log in a perfect mathematical process? It will be satisfactory to compute the length of a spiral line that represents the position of the cutting blade. This requires use of the integral calculus.

Unbelievably, the more formulas you write, the less some people believe you. So, the second question is this; can you think of a simpler method of estimating the length of the sheet of wood that appeals to common sense and involves nothing more than arithmetic and geometry?

Solutions to "The Spiral Puzzle"

If we think about the spiral we are describing as proceeding from the center outwards, then every time it completes a revolution ($2*\pi$ radians), the radius of the spiral has increased by 1/4 inch. Therefore, the appropriate spiral formula relating **r**, the distance from the center, to θ, the angle of revolution, is

Since $L = r * \theta$; $dL = r\, d\theta$; but $r = ¼ * \theta / (2*\pi)$; therefore $dL = \theta / (8*\pi)\, d\theta$

Next, we observe that the number of revolutions will be 48 per radial foot. Since each revolution advances us 1/4 inch from the center, and we have to advance a total of 2 feet to reach the outer surface, the number of revolutions is 96. Hence, the length of the spiral is:

$$\text{Length} = \int_0^{96 * 2 * \pi} \theta / (8 * \pi)\, d\theta = 1 / (8 * \pi) * \int_0^{192 * \pi} \theta\, d\theta$$

$$= 1 / (8 * \pi) * \left(½ * \theta^2 \Big|_0^{192 * \pi} \right)$$

which gives us

$$= 1/(8 * \pi) * (½ * (192 * \pi)^2 - 0)$$

$$= 192^2 * \pi / 16 \text{ inches}$$

or

$$\text{Length} = 2304 * \pi \text{ inches} = 192 * \pi \text{ feet} = 603.1857\ldots \text{ feet approximately.}$$

Here, we are assuming that the length of the spiral is a good estimate of the length of the sheet. Of course, a spiral is linear like a string, while a sheet has thickness, and may be inherently curved, with the outer side being longer than the inner side.

Now if the manager doesn't believe all your high-falutin' math, you can simply lean back in your chair and ignore him, secure in the knowledge that you are right; or you can repeat everything you said but **LOUDER;** or you could try a different approach to the problem that allows your audience to follow you more easily.

A reasonable way of estimating the behavior of the problem is to suppose that, if we have a perfect cut, then all the wood in the log goes into the wooden sheet, and the curvature of the wooden sheet can be ignored. The resulting sheet can be viewed as a 10-foot wide, ¼-inch high and "very long" box. To determine its maximum theoretical length we ignore the cutting process, and pretend that the whole log was simply "melted" and poured back into a box-shaped mold.

Now, we simply note that the volume of the log must equal the volume of the box, so, remembering that 1/4 inch = 1/48 foot, we have

$$(\pi * r^2 * W)\ ft^3 = \text{Volume of Cylinder} = \text{Volume of Box} = L * W * H\ ft^3$$

$$\pi * r^2 = L * H$$

$$\pi * 2^2 = \text{Length} * 1/48$$

which tells us that

$$\text{Length} = 4 * \pi * 48 = 603 \text{ feet (approximately)}$$

Using the formula on page 4 of the lab handout gives the same result. Try it, assuming the inside radius is zero with an outer radius of 2 feet. Go ahead, give it a whirl.