Lesson 00-A1-Exponents

Exponents are a concise way of expressing multiplication and roots. A square with sides of length a has an area of $a \times a$, which is more concisely written as a^2 (a raised to the second power). This is often read as "a squared." Similarly, a cube of side a has a volume of $a \times a \times a$, which is more concisely written as a^3 (a raised to the third power) or "a cubed." In general, a raised to the nth power is a^n, where the exponent n can be any real number, fractional or whole, positive or negative.

Quantities raised to various powers often appear in equations where they may be multiplied or divided. Thus, $a^2 \times a^3 = (a \times a \times a) \times (a \times a) = (a \times a \times a \times a \times a) = a^5$. This is a general result, such that

$$(a^n)(a^m) = a^{n+m}$$

When the base numbers (a in this case) are the same, the exponents add when you multiply. Alternatively, when the base numbers are different but the exponents are the same the bases are multiplied

$$(a^n)(b^n) = (a \times b)^n$$

Division with exponents has a similar set of rules. Thus $a^3/a^2 = (a \times a \times a)/(a \times a) = a$. This is a general result. When the base numbers are the same, the exponent in the denominator is subtracted from the exponent in the numerator. Thus, as a general rule

$$a^n/a^m = a^{n-m}$$

This suggests another frequently encountered definition,

$$1/a^m = a^0/a^m = a^{0-m} = a^{-m}$$

For example, $1/5 = 5^{-1}$; $1/2^2 = 2^{-2}$; $1/7^{-2} = 7^{-(-2)} = 7^{+2}$. Since we know that $a/a = 1$, it follows in this notation that $a / a = a^{1-1} = a^0 = 1$. In fact, any number, quantity or variable raised to the zero power equals 1. Here are some other simple rules governing arithmetic using exponents.

Fractional exponents are roots: $a^{1/n} = \sqrt[n]{a}$; $\sqrt{a}\sqrt{a} = a^{½}a^{½} = a^{½+½} = a^1 = a$

Powers undo roots and vise versa: $(a^{1/n})^n = a^{n/n} = a^1 = a$. In general, raising an exponent to a power is accomplished by multiplying the exponents together.

Since we know that $(25^{½})^2 = (5)^2 = 25$. It follows as a general rule that $(a^n)^m = a^{nm}$, where n and m can be whole or fractional, positive or negative.