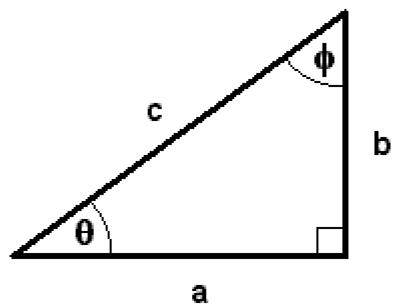
Lesson 00-C-Trigonometry

The trigonometric functions are simply ratios of the sides of a right triangle. At least that is one way to look at them. The trigonometric functions of most interest to us are known as sine, cosine and tangent; abbreviated sin, cos and tan.

Classical Introductions – The trigonometric functions are usually introduced into mathematics classes with reference to a right triangle with sides labeled a and b and a hypotenuse labeled c, as in the figure.



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{b}{c} \qquad \sin\phi = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

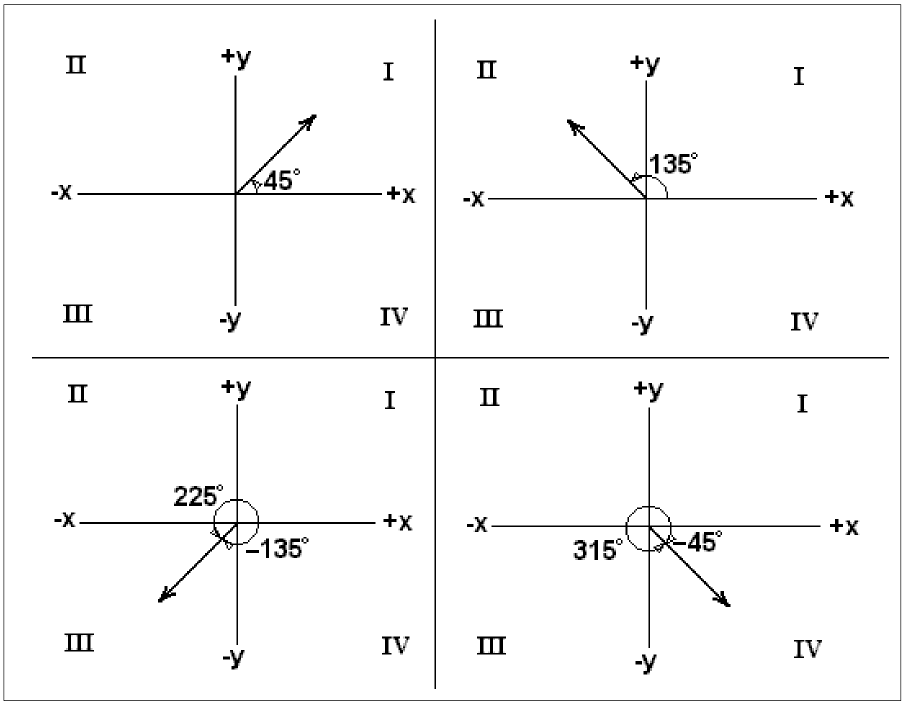
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{c} \qquad \cos\phi = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{b}{a} \qquad \tan\phi = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

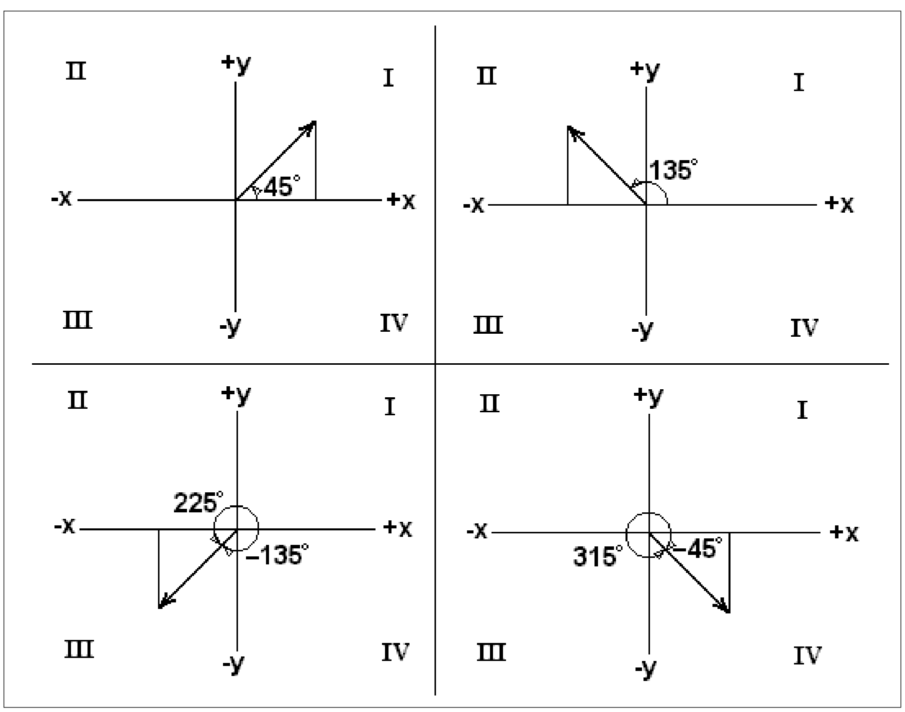
This works well enough for an introduction, but it seems to put most of the emphasis on the properties of the triangle. The most important and persistent use we make of the trigonometric functions is our work with vectors. Vectors are viewed like arrows, lines pointing in a specific direction, and we will focus on the angle that indicates the direction. We need to view the trigonometric functions with an eye on the angles rather than on the triangle.

A vector is just an arrow, with an arrowhead, the head, and a tail. It points in a specific direction and we need good ways to represent that direction mathematically. This is where the trigonometric functions come into their own. First, we use the angle relative to a reference frame attached to the tail of the vector. Then we use the trigonometric functions to calculate the "components" of the vector that point along the main axes of the coordinate system.

In the plane we need only two perpendicular axes. One will usually be horizontal across the page and the other will usually be vertical up-and-down the page. The axes break the plane into four quadrants. Here are examples of vectors in each of the four quadrants. Quadrants are numbered with roman numerals from I to IV.



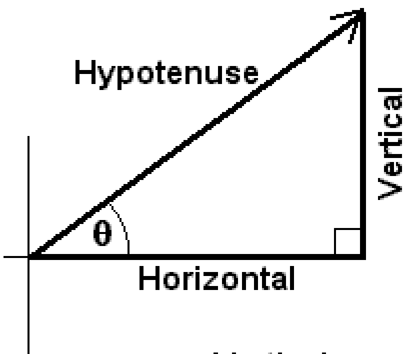
Note that the vectors in quadrants III and IV are equally well measured with negative as with positive angles. The vector serves as the hypotenuse for defining the trigonometric functions. To find the sides of the right triangle, drop a perpendicular line from the head of the vector to the x-axis. This is important: do **not** draw the first line to the y-axis.



Here is the revised diagram showing the right triangles used for defining the trigonometric functions on a vector.

This time we will define the ratios using the vertical side and the horizontal side of the triangle. For this system to work, however, you must have the triangles drawn as shown here. The x-axis always contains the horizontal side.

When working with vectors we will define the trigonometric functions as follows:



$$\sin\theta = \frac{\text{Vertical}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\text{Horizontal}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Vertical}}{\text{Horizontal}}$$

The height of the vertical side above or below the x-axis is called the y-component of the vector. The width of the horizontal side left or right of the y-axis is called the x-component of the vector. The length of the hypotenuse is called the magnitude of the vector. The angle θ is the direction of the vector. See the separate handout on vectors for more details about vectors.

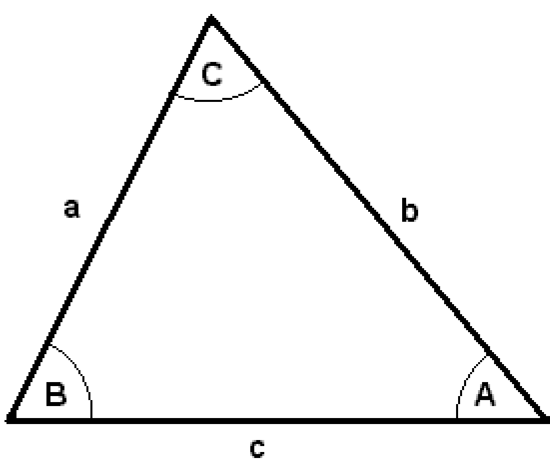
The trigonometric functions allow us to incorporate the directional information about vectors into the mathematics in a simple and logical way. Every time you see a term like $\sin\theta$ in an equation, you know that directional information has been included in the mathematical derivation of that expression.

The mathematics of the trigonometric functions is rich and interesting in its own right, but we have rather specific uses for these functions.

There are a large number of relationships among the trigonometric functions, but here is the rather small list that you will want to memorize. These you will find useful.

$\tan\theta = \sin\theta / \cos\theta$ $\sin^2\theta + \cos^2\theta = 1$ $\sin 2\theta = 2\sin\theta\cos\theta$

The trigonometric functions are also useful when working with any triangle, not just with right triangles. Here are two more relationships you should commit to memory.



Law of Sines $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of Cosines $c^2 = a^2 + b^2 - 2ab\cos C$