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Physics: An Incremental Development, John H. Saxon, Jr.

Lesson 01 – Units, Scientific Notation and Significant Digits

Student Notes:

Numbers

Certain **theoretical numbers** are exact – no error – infinite number of significant figures.

Circumference = π • diameter; no error in π

Kinetic energy = K = ½ m v^2; no error in ½

Certain **defined unit relationships** are exact – no error – infinite number of significant figures. Any additional digits after those given are all zeroes.

12 inches = 1 foot;

1 inch = 2.54 centimeters,

1000 grams = 1 kilogram & many more

Certain physical constants are also defined. Some of these may seem as though they should be measured, but in fact they are defined to have exact values. All digits after those listed are zeroes. The four are:

Coulomb constant = $k_e = 8.987\ 551\ 787 \times 10^9$ N•m²/C²

Permeability of free space = $\mu_0 = 4\pi \times 10^{-7}$ T•m/A

Permittivity of free space = $\varepsilon_0 = 1/\mu_0 c^2 = 8.854\ 187\ 817 \times 10^{-12}$ $C^2/N \cdot m^2$

Speed of light in vacuum = $c = 2.997\ 924\ 58 \times 10^8$ m/s

Most measured physical constants have very high precision, more significant figures, than any of your measured numbers. Not infinite, but many significant figures. Digits after those give here are unknown. Examples include:

Avogadro's number = $N_A = 6.022\ 136\ 7 \times 10^{23}$ particles/mol

Electron mass = $m_e = 9.109\ 389\ 7 \times 10^{-31}$ kg

Gas constant = R = 8.314 510 J/K•mol

Neutron mass = $m_n = 1.674\ 928\ 6 \times 10^{-27}$ kg

Proton mass = $m_p = 1.672\ 623 \times 10^{-27}$ kg

Measured numbers

These always have some error because of limitations in the scale of the measuring instrument. By default, the last (least significant) digit in any measured number is found by estimating to the nearest 1/10th of the distance between the finest gradations on the scale. The readability of the scale determines what we call the precision of the measurement. For example, the precision of the meter sticks available in the lab is 0.1 mm = 0.01 cm = 0.0001 m

In other cases, the least significant digit in the measurement is found using the manufacturers stated precision for the instrument. For example, if the instrument has a stated precision of ±0.01, then a measurement of 3.03 has 3 significant figures. A measurement of 27.36 on the same instrument has 4 SF. The stated precision indicates the location of the last significant digit, not the number of significant figures in the values measured with the particular instrument.

While precision tells us how closely the scale can be read, the accuracy tells us how close the measurement comes to getting the right answer. Imagine a scale on an instrument that is very finely divided and thus has high precision. If that instrument is poorly designed, the highly precise numbers may yet be very inaccurate if the results of the measurements are far from the correct answer.

Scientific Notation:

$4,221,000 = 4.211 \times 10^6$ $0.000\ 004\ 221 = 4.211 \times 10^{-6}$

Using measured values, including their errors, in your calculations:

If a measured number is included in a calculation or series of calculations then the final result cannot have more significant figures than the smallest number of significant figures in any of the measured numbers used in that calculations.

If performing a series of calculations do not reduce the intermediate answers to the correct number of significant figures. Do that only once, at the end of the calculations.

It is a good practice, after finishing a long calculation, to go back an cross out the excess digits in the intermediate calculation using a single horizontal line. Don't obliterate the extra digits; you may need to refer to them again.

Rules for reading and for reporting significant figures

Reading

1) All non-zero numbers must be significant – otherwise they wouldn't have been included.

   3.4567 (5 SF)

2) All internal zeros, between non-zero numbers, are significant.

   3.405 (4 SF)   340.5 (4 SF)   3405 (4 SF)

3) For values less than one, place-holding zeros to the right of the decimal point (*to the left of the first non-zero digit*) are not significant.

   0.0345 (3 SF)   0.00000345 (3 SF)   0.0000000000000000345 (3 SF)

4) Trailing zeros after the decimal point are significant, otherwise they wouldn't have been included. If trailing zeros are shown after the decimal point, you must assume they meant these numbers to be read as significant. *This is one of the most common errors that beginning students make. Don't forget to include the trailing zeros when they are significant. Every time you make a measurement you need to be* <u>*thinking*</u> *when you record the number in your data table. Include the correct number of significant figures even when some of those significant digits are zeros. If you want to keep extra digits in your data tables for use in later calculations, lightly line them out so they can be read but will not be interpreted as significant digits by an unwary reader, or even by a wary one.*

   0.003450 (4 SF)   0.00345000 (6 SF)   0.00345~~00~~ (3 SF)

5) Trailing zeros to the left of the decimal point on numbers greater than 1 are assumed to be merely placeholders (*not significant figures*) unless additional information is provided to indicate where the significant figures end.

   There are several ways to satisfy this last rule. In our textbook and in this course we will sometime use either an overscore or an underscore, when necessary, to indicate the last significant figure. An even better method, which avoids all confusion, is to report results in scientific notation. In scientific notation all the digits are significant figures. So when you use scientific notation, include all the significant digits in the answer and don't include any digits that are not significant.

   1,234,000 (uncertain; is it 4 SF, 5 SF, 6 SF, or 7 SF; assume 4 SF without more info)

   1,234,$\overline{0}$00 (5 SF) or 1,234,$\underline{0}$00 (5 SF)   or 1.2340×10^6 (5 SF)

   1,234,0$\overline{0}$0 (6 SF) or 1,234,0$\underline{0}$0 (6 SF)   or 1.23400×10^6 (6 SF)

Units & Unit Conversions & Unit Conversion Factors

Unit conversion factors always begin with an equality relating two units of the same type.

Time units	Length units	Length units
1 minute = 60 seconds;	12 inches = 1 foot;	1 inch = 2.54 centimeters

Because these are always made up of defined values, there is no error in these numbers.

Each equation creates two unit conversion factors. The word "unit" is a sort of scientific pun. It refers to the fact that we are using units and also to the fact the each unit conversion factor is equal to unity, i.e. 1. Thus,

1 = 1 minute/60 seconds and 1 = 60 seconds/1 minute

1 = 12 inches/1 foot and 1 = 1 foot/12 inches

1 = 1 inch/2.54 centimeters and 1 = 2.54 centimeters/1 inch

Since a unit conversion factor is "equal to" one, we can multiply by such a conversion factor without changing the underlying quantity. Whether we call a certain length 12 inches or 1 foot, we are still talking about the same distance. The numbers and units are different but the same quantity of distance is clearly indicated by both.

We will spend part of our time in this course learning about units and there is a separate handout on units so we do not need to repeat all that work here. There is one point you must remember, however. Any answer that you report, in lab, on quizzes, on homework must include the unit(s) with the numerical answer. A number alone, without the units, stands for nothing, means nothing, and is worth nothing.

Unit conversions can be complex and you will get lots or practice during the year. One special case of particular note is worth mentioning right up front. Anytime you convert between SI units of length and "American Customary" units of length, you must use the conversion mentioned above; 1 inch = 2.54 centimeters.

To convert 30.0 yards to meters, the following steps are required. There are no optional steps.

$$30.0 \text{ yards} \times \frac{3 \text{ feet}}{1 \text{ yard}} \times \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{2.54 \text{ centimeters}}{1 \text{ inch}} \times \frac{1 \text{ meter}}{100 \text{ cm}}$$

= 27.4~~32~~ m (3 SF) = 27.4 m

To convert 17 miles per hour to meters per second, here are the steps.

$$\frac{17 \text{ miles}}{1 \text{ hour}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} \times \frac{1 \text{ meter}}{100 \text{ cm}} \times \frac{1 \text{ hour}}{3600 \text{ sec}}$$

= 7.5~~9968~~ m/s (2 SF)

= 7.6 m/s (2 SF)

Rounding Numbers

Rounding of numbers is only used for display purposes. It is used to display an answer in such a way that it is obvious to the informed reader how many significant figures are appropriate at this particular stage of the calculation. Under no circumstances are rounded numbers ever to be used in a continuation of the calculation. Always use un-rounded numbers in your follow-on calculations.

Typically, you will need to redisplay an answer in rounded form only at the end of a series of calculations and, perhaps, at one or two selected points in a series of calculations.

Again, I repeat, do not use the rounded version of the answer to carry on a series of calculations. Always write down both the unrounded and the rounded answers so that you have both clearly recorded on homework, quizzes and labs.

The unrounded answer does not need to include all the digits displayed on your calculator. That is overkill. What you need to keep in the unrounded answer when you write it down is two or three more digits than the current number of significant figures appropriate to your eventual result. That provides enough extra digits to insure that you will not degrade the future calculations or add calculation-based errors to the result. Write those extra digits in the unrounded answer even if they happen to be zeroes and strike through them after completing all the calculations.

Determining the correct number of significant figures for your answer

Use the highest numbered rule that applies to your particular situation. Sometimes, different rules apply to each segment of a series of calculations. Keep checking as you go through each step of a series of calculations to make sure you are using the right rule at each step.

Rule 1: Assume all answers have three significant figures. (*When you don't know anything about the measurements and were not involved in making the measurements.*) This rule applies to WebAssign assignments unless the number of significant figures in your answer is specified in the question.

Rule 2: Assume the answer has the same number of significant figures as the measured number with the smallest number of significant digits. (*Notice that measured numbers are the ones to look at. It is important that when you write down your measured values, you write them showing the correct number of significant digits appropriate to the precision of the measuring device. It you get it wrong at the beginning it will always be wrong.*)

Rule 3: The answer might have fewer significant figures that the measured numbers, if two numbers with very different magnitudes where added or subtracted during the calculations. If only multiplication and division are used, there should be no problem.

Rule 4: When taking trigonometric functions or logarithmic/exponential functions – there is no simple rule you can apply without using calculus. To be on the safe side, always keep at least 5 digits in your answer. This way we let the other measured numbers determine the significant figures.

Challenge Questions – In addition to those in the textbook

1.21 A newton is a kg m / sec^2. Indicate which of the following unit(s) is/are equivalent to one newton?

a) 1000 gram centimeters / sec^2

b) 0.001 kg kilometers / sec^2

c) 60 kg m / min^2

d) 3600 kg m / min^2

e) 1 E5 grams cm / sec^2

f) 3.6 E9 gram mm / min^2

g) 1 joule per meter (*a joule is a newton•meter*)

1.22 What is the sum of 306 mm + 1.22 meters plus 7.4 E5 nanometers plus 0.0033 km plus 44.2 cm?