Lesson #2

2A Estimating with Scientific Notation
2B Precision of a Calculation
2C Unit Conversions

Estimating:

The only rules are to round viciously (to one digit); don't lose track of the exponents.

It helps to practice this all the time. You will develop a personal style. That is; personal habits that work for you and during which you do not make mistakes. This can be very individual and idiosyncratic. Practice writing the estimating process and very gradually, over the weeks, start to do more and more in your head. Most of us never are quite able to do all of it all mentally. There are too many pitfalls where mistakes can be made. But it can still become a quick and reliable way to estimate the answer as a check on how well your longer and more time consuming calculation worked.

Example 1: [correct answer = 1.01885×10^{-26} = SF = 1.02×10^{-26}]

Given the problem to estimate $$\frac{(0.000344 \times 10^{-6})(4410)}{(0.00623 \times 10^{15})(2390 \times 10^{4})}$$

First, round to single digit whole numbers in all pre-exponential terms

$$\frac{(3 \times 10^{-4} \times 10^{-6})(4 \times 10^{3})}{(6 \times 10^{-3} \times 10^{15})(2 \times 10^{3} \times 10^{4})}$$

Pull out the pre-exponential terms and do the math

$$\frac{(12 \times 10^{-4} \times 10^{-6} \times 10^{3})}{(12 \times 10^{-3} \times 10^{15} \times 10^{3} \times 10^{4})}$$

Add the exponents together

$$\frac{(12 \times 10^{-4-6+3})}{(12 \times 10^{-3+15+3+4})}$$

$$\frac{(12 \times 10^{-7})}{(12 \times 10^{+19})} \equiv 1 \times 10^{-26}$$

Example #2

Given (16)(5280)(5280)(5280)(12)(12)(12)

Round to one digit, count powers of 10, and multiply sig figs

Powers of 10	1 + 3 + 3 + 3 + 1 + 1 + 1 = 13
	20 x 5000 x 5000 x 5000 x 10 x 10 x 10
Single digits	2 x 5 x 5 x 5 x 1 x 1 x 1 = 250

Collect the terms appropriately

$$250 \times 10^{13} = 2 \text{ or } 3 \times 10^{15}$$

This is quick and dirty verification, only. It is not a substitute for the full blown calculation. The actual result is 4.1×10^{15}. The estimate is performed primarily to confirm that the full calculation was done correctly.

Precision vs Accuracy:

Precision only means that your repeated measurements agree with each other to a satisfactory degree.

Highly precise measurements may still be very inaccurate if --

- your instruments are not properly calibrated
- you misread the output or misuse the instrument
- you don't control all the variables

Precise measurement tools do not guarantee accurate results. The experimenter always plays an important role in accuracy. The precision of the tool only becomes the limiting factor on the accuracy of the experimental result, if the tool is properly calibrated, the tool is used correctly, and the experimenter does everything properly. That is one of our goals as experimentalists; to use the equipment properly and to its fullest potential without overstating or understating its precision or the accuracy of our results.

Unit conversions:

Having made a measurement relative to one standard of measurement, it is then possible to calculate what the result would be if it had been made relative to a different standard of measurement. Note that the measured item is not changed by this operation You will get a different number with different units attached. That is all. Only the reference standard is being changed, not the physical item or event that was measured. This is only possible to do with high confidence because there are so many well defined reference standards and their relationships to one another are by now well established or well defined.

First, do not try to change between fundamentally different units. You may convert from one unit of length to another unit of length (inches to feet, perhaps), but you cannot change a length measurement into a time measurement. Length and time are entirely different things (almost). Until we get to relativity, don't try converting between what we ordinarily think of as fundamentally different measurable quantities.

Unit conversion factors are called "unit" conversions because

1) they allow us to change from <u>one standard unit</u> to another.

2) because they are by definition <u>equal to unity</u> (= 1.000000...).

Unit conversions always start with one or more equalities. Preferably internationally verified and agreed equalities where both sides of each equation are defined to an arbitrarily high degree of accuracy (note accuracy). For example, there are exactly 12 inches in a foot, exactly 100 centimeters in a meter, exactly 2.54 cm in an inch, etc.

Being a citizen means that you are expected to know all of the following equalities which can serve as the basis for unit conversion factors (your textbook and most dictionaries have a table somewhere full of these if you need more examples):

12 inches	=	1 foot
3 feet	=	1 yard
5280 feet	=	1 mile
1760 yards	=	1 mile
60 seconds	=	1 minute
60 minutes	=	1 hour
24 hours	=	1 day

In addition, you must know from the SI prefixes, things like the following"

100 cm	=	1 m
1000 grams	=	1 kilogram

[See the table of SI multipliers inside the back cover of the textbook.]

There is one new relationship that you must now memorize. It is the only one that will allow you to convert between U.S. Customary units and SI units of length. It is

2.54 cm	=	1 inch

All length conversions from SI to U.S. Customary, or vice versa, must include this one somewhere along the way.

Given any one of these equations, you can construct two conversion factors (each is the inverse of the other), both of which are by definition equal to unity. Taking that last one for example, we can divide both sides by 1 inch to get

1.0000...	=	2.54000... cm / 1.0000... inch

or we can divide both sides by 2.54 cm and multiply by 1.00000 inches to get

1.0000...	=	1.000... inch / 2.54000... cm

When attempting complex unit conversions it is important to first perform a dimensional analysis. This will help you figure out which form of each conversion factor to use so that you can be sure the units all cancel properly. It will save time in the long run.

Example:

To convert <u>317 miles to nanometers</u>, you would start as follows with a dimensional analysis that you can use as a template to work out the full solution later. Here is a little technique that makes this a little easier for you. Cross out lightly so that you can still read the units otherwise you may not remember where to put all the numbers.

$$\text{miles} \times \frac{\text{feet}}{\text{mile}} \times \frac{\text{inches}}{\text{foot}} \times \frac{\text{cm}}{\text{inch}} \times \frac{\text{m}}{\text{cm}} \times \frac{\text{nanometers}}{\text{meter}} = \underline{\qquad\qquad} \text{ nanometers}$$

Once you've verified that all the units cancel properly, then fill in the numbers and start working out the math.

$$317\ \text{miles} \times \frac{5280\ \text{feet}}{1\ \text{mile}} \times \frac{12\ \text{inches}}{1\ \text{foot}} \times \frac{2.54\ \text{cm}}{1\ \text{inch}} \times \frac{1\ \text{m}}{100\ \text{cm}} \times \frac{10^{9}\ \text{nanometers}}{1\ \text{meter}} = \underline{5.10 \times 10^{14}}\ \text{nm}$$

Example:

Now determine the unit conversion factor, and its inverse, that will take you directly from feet to meters.