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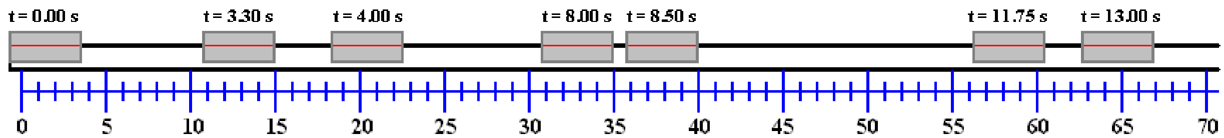
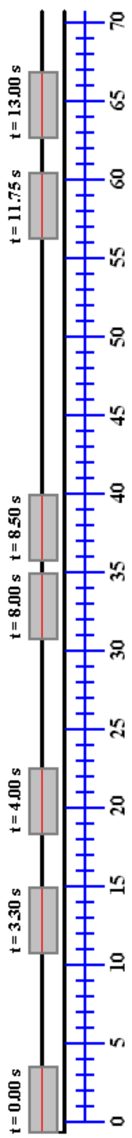
Physics: An Incremental Development, John H. Saxon, Jr.

Lesson 07 – Linear Motion, Average Velocity, Average Acceleration

Reference Frames - We begin by examining linear motion. It is simply motion in a straight line; any straight line. Here is one example of a cart observed by two different observers. Each diagram shows the same object moving along the same track. The only difference is the "apparent" direction. By convention one observer will say that this cart above is moving in the positive direction along the along the x-axis. The cart moves from left to right from this observer's point of view. Both will agree that this is straight-line motion.





In the diagram to the left the same cart is moving along the same track. It is making exactly the same trip it made in the first diagram. By convention a second observer would say that it is moving in the positive direction along the y-axis. The cart moves from bottom to top from this observer's point of view.

Both observers are correct. The first observer will report motion along the x-axis in her reference frame. The second observer will report motion along the y-axis in his reference frame. To compare their results they need find a common reference frame.

In practice, we will use a common reference frame known as the laboratory frame. It can be chosen arbitrarily for our convenience from one laboratory experiment to the next. We can call the axes anything we like, but it is conventional to use "x" for horizontal motions that move across the laboratory, and "y" for motion that moves vertically up-and-down during the experiment.

The only thing special about the laboratory reference frame is that everyone understands in advance what it is and agrees to use the same reference frame. If you ever have doubts about what constitutes the current laboratory reference frame in our laboratory assignments, you should ask for clarification.

In homework problems it is a little trickier, perhaps. Homework problems are not worked in the world; only in your head. So, it may seem that you have much more latitude to pick a reference frame. However, we will all agree to call motion that is left-and-right on the page or on the board motion along the x-axis or motion along the East-West axis. Also, we will all agree to call motion that is up-and-down on the page or on the board motion along the y-axis or motion along the North-South axis. This is our equivalent of a laboratory reference frame for homework problems.

We will refer back to these two diagrams in the sections that follow.

Linear Motion – There are a several ways to show linear motion. One way is to watch an object move in a straight line. A visual observation is one type of experimental record. You might even record the motion so you can replay it later. In science, however, there is nothing quite as useful as numerical data. With numerical data in hand you are ready to try to understand processes. A visual record is just not data of a type that we find very useful very often, although there are exceptions.

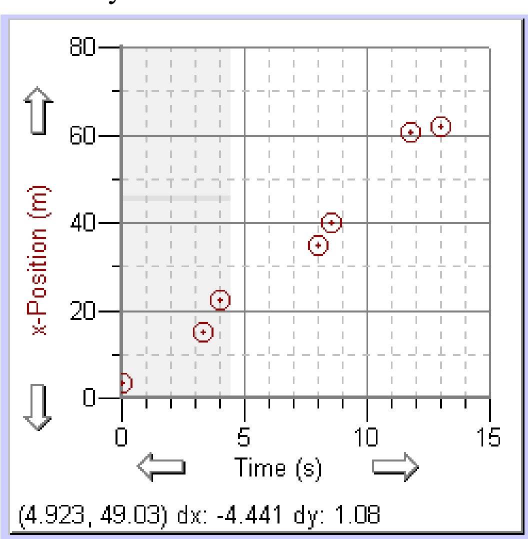
The next best way to show linear motion is with diagrams like those on the previous page. The diagrams show the position and the time at each position as the cart moves along its track. These diagrams look very much like cartoons of actual experiments and are instructive for that reason as well. A well-made and well-labeled diagram is often an essential part of a complete laboratory report.

Another very good way to show linear motion is with a data table. We can get the numerical data from a visual record, from a diagram like those on the previous page, or from a measuring device. We'll do one table for the horizontal diagram and a second table for the vertical diagram. We will use the location of the front edge of the cart as our marker for the cart position in both cases. We could use any other arbitrary point on the cart, but the front of the cart provides a convenient edge for reading the position scale.

Time (s)	x-Position (m)
0.00	3.5
3.30	15.0
4.00	22.5
8.00	35.0
8.50	40.0
11.75	60.5
13.00	62.0

As you can see, since both observers were watching the same cart, the data looks identical except for the labeled axes names. Clearly, the name you assign to an axis is not nearly as important as the process you are measuring with that axis. We'll use position along the x-axis from now on in this analysis.

Time (s)	y-Position (m)
0.00	3.5
3.30	15.0
4.00	22.5
8.00	35.0
8.50	40.0
11.75	60.5
13.00	62.0



Perhaps the best way to show linear motion, indeed the best way to present most processes, is with a graph. The graph must be generated from a data table like those above. A graph showing only the data points is an honest representation of the facts known about the motion. What we want is to understand is, "How was the cart moving?"

If you were there, or if you have a visual record, then this would be a good time to consult your memory or the video. We need a clue about what was happening between the measurements before we draw any kind of line. Even the so called connecting lines are misleading when we have no idea of what was really going on.

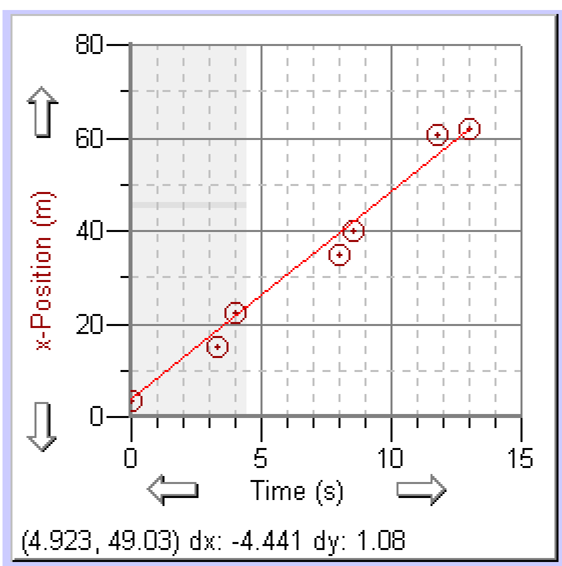
We don't have it. We weren't there. Without additional information, what can we say or do? We have a sample of the instantaneous position information; nothing more. Beyond that we can only report on what is in that data. Beyond those few instantaneous positions, we can also report the average values of velocity and acceleration; $\mathbf{v}_{AVE}$ and $\mathbf{a}_{AVE}$.

Average Velocity – Any average motion must be calculated over a finite time interval. Usually, it is essential that you list both the starting time, t_i, and the ending time, t_f, of that interval. From these you can calculate the time difference, $\Delta t = t_f - t_i$. This is just the type of information that can be extracted from our data table.

Graphically, finding the average is like drawing straight lines between the two data points. We could use the connecting lines in Graphical Analysis, if all we wanted was the averages between consecutive time points. What if we needed the average for the whole trip? What if we needed the average between t = 4.00 s and t = 11.75 s? In those cases we need to draw a straight line from the point at t_i to the point at t_f.

The straight line is not meant to imply that the points along the line represent actual locations of the cart at those intermediate times. Rather, they represent points where the cart would have been **IF** it had been moving at constant velocity during the time interval. Calculating an average is what you do when there is not enough information. If we knew where the cart was at each instant, we would not need to calculate the average. We could, but we might just as well report the true location at any instant.

What are we going to do with this line between two data points? Let's try an example. We'll look at the average for the whole trip. Here is the graph again with the appropriate line drawn in red for clarity.



The red line shows "the average" position the cart would have if it moved at constant speed from its starting location to its ending location in the same amount of time.

The red line is interesting. A couple of times the cart came very near to being on this line even though its actual motion seems to have been a little more complicated.

Another interesting aspect of this line is its slope. As you should know, the slope of a line on any graph is the change in the vertical axis value divided by the change in the horizontal axis value. In terms of our variables on this particular graph, this means

$$\text{Slope} = (x_f - x_i) / (t_f - ti) = \Delta x / \Delta t = \text{velocity in m/s}$$

Look in the data table for the values to substitute into this expression:

Slope = (62.0 – 3.5) / (13.00 – 0.00) = 58.5 / 13.00 = 4.50 m/s

What about the units? The numerator is in meters and the denominator is in seconds, so the units of the slope must be meters per second, m/s. That means slope is velocity. It has to be!

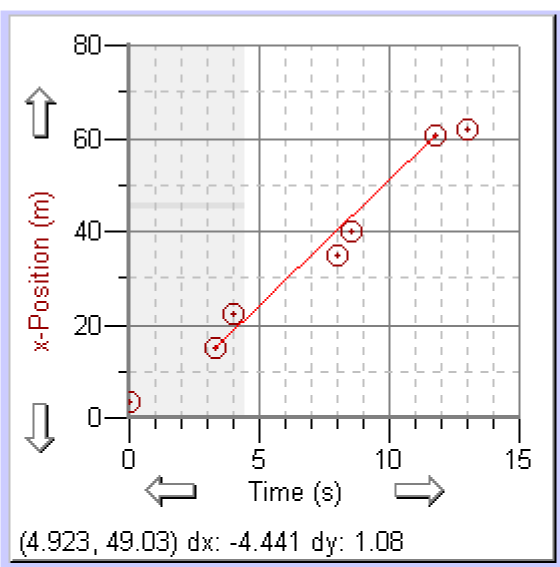
You must be careful. The slope of every possible graph is not a velocity. BUT, the slope of EVERY position versus time graph is a velocity. The meaning of the slope depends on what you graph on the axes. Any change in the graph will change the meaning of the slope. Even if all you did was plot the time vertically and the position horizontally, the slope would no longer be velocity. So, again, be careful.

Slope is velocity on a position vs time graph and ONLY on a position vs time graph.

Clearly, what the slope is telling us is how fast the cart would have to move on average in order to make the same trip in the same amount of time at constant speed. Notice that the slope is constant over the entire time interval. That means we are calculating the constant velocity required to complete the trip in the same amount of time. The actual velocity appears not to have been constant. What have here is the average of all the instantaneous velocities along the way.

Specifically, what we have calculated from the slope in this case is the AVERAGE VELOCITY of this particular trip. We've calculated that the average velocity of the entire trip was 4.50 m/s. It tells us nothing about the true velocity at any instant. It only tells us about the average velocity of the cart during this specific time interval.

We can do the same thing for any other segment of the trip. All we need to know are the starting and ending positions and the starting and ending times.



For example, what is the average velocity of our cart between time t = 3.30 s and time t = 11.75 s?

Slope = (60.5 – 15.0) / (11.75 – 3.30) = 5.38~~46~~ m/s

This is clearly a different average velocity. Well, it's over a different time interval, too. The cart was obviously moving at different speeds at different times. So, it is not too surprising that when we look at different intervals we get different averages.

The highest average velocity that occurred during this trip occurred in the interval between 3.30 s and 4.00 s. The lowest average velocity occurred in the interval between 11.75 s and 13.00 s.

Calculate the average velocities over each of these time intervals?

v_{AVE} (t=3.30 s to t=4.00 s) = ____________________________ m/s

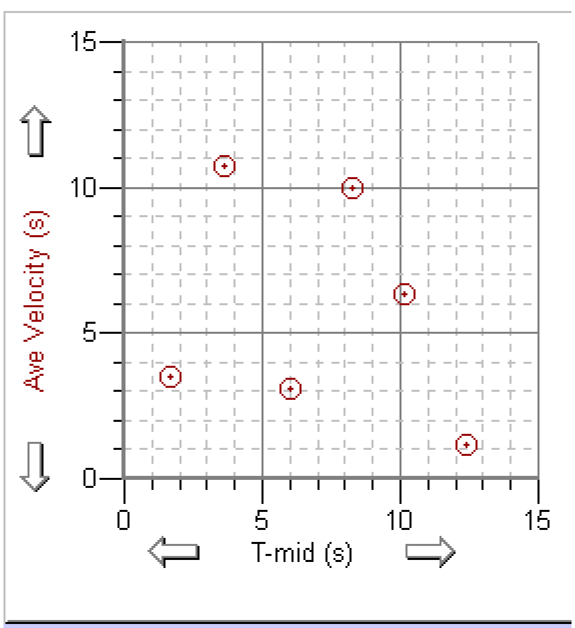
v_{AVE} (t=11.75 s to t=13.00 s) = ____________________________ m/s

Average Acceleration –

Time (s)	x-position (m)	Interval Mid-Time (s)	Average Velocity m/s)
0.00	3.5		
3.30	15.0	1.65	3.484848
4.00	22.5	3.65	10.7143
8.00	35.0	6.00	3.125
8.5	40.0	8.25	10.0
11.75	60.5	10.125	6.30769
13.00	62.0	12.375	1.20

To get the average acceleration we need to expand our data table with some additional calculations. This time we need the average velocity for each leg of the trip. We will plot these at the mid-point time of each leg so include those mid-point times in the table, as well. Acceleration is a measure of how fast the velocity is changing. As usual, when we take an average, we will calculate the acceleration required to make the same velocity change in the same amount of time.

Use the two new columns to create a new graph; a velocity vs time graph this time.



This graph shows the average velocity of each leg of the trip. The average velocities are plotted at the mid-point of each leg – in the spirit of the average. The velocity was changing from just below 5 m/s to just above 10 m/s until the end where the cart appears to be slowing to a stop. We don't know what happened next; it just looks as though it might be about to stop.

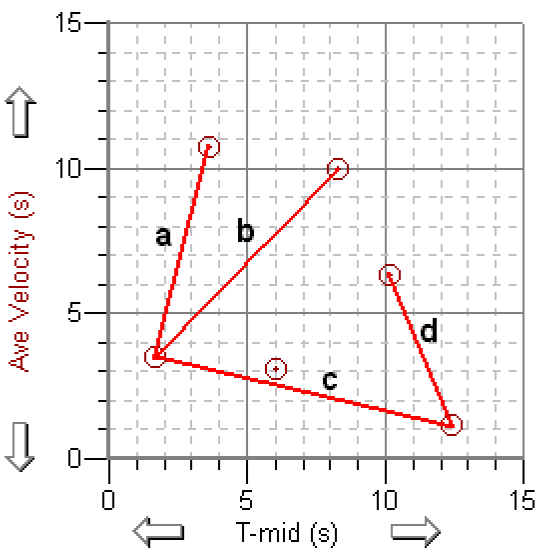
Like the average velocity, the average acceleration is computed over a time interval. Between the first and second legs, the cart accelerated. Between the second and third legs the cart decelerated.

The average acceleration is the constant acceleration the cart would need to maintain to make the same velocity change in the same time.

If we check the units for the slope of a straight line on a velocity vs time graph, we see the slope has the correct units for acceleration. Here is the slope formula

$$\text{Slope} = (v_f - v_i) / (t_f - t_i) = \Delta v / \Delta t = \text{acceleration in m/s}^2$$

The definition of slope and average acceleration are the same and the units of the slope are the units of acceleration. The slope of a velocity vs time graph can only represent an acceleration. Because the velocity and acceleration both seem to be changing during the trip, we will get a different average acceleration for each interval we check.



The graph at the left shows four possible intervals that we could check. Two of these have positive slopes indicating acceleration. Two of them have negative slopes indicating deceleration.

These randomly chosen intervals are:

a) a_{ave} between 1st and 2nd legs.

b) a_{ave} between 1st and 4th legs.

c) a_{ave} between 1st and last legs.

d) a_{ave} between the two final legs.

Get the actual numbers from the data table.

Calculation of average acceleration

We number the legs 1 through 6 (from first to last). Here are the calculations.

a) $a_{ave} = \text{slope} = (v_2 - v_1) / (t_2 - t_1) = (10.7143 - 3.484848) / (3.65 - 1.65) =$ +4.21~~47~~ m/s²

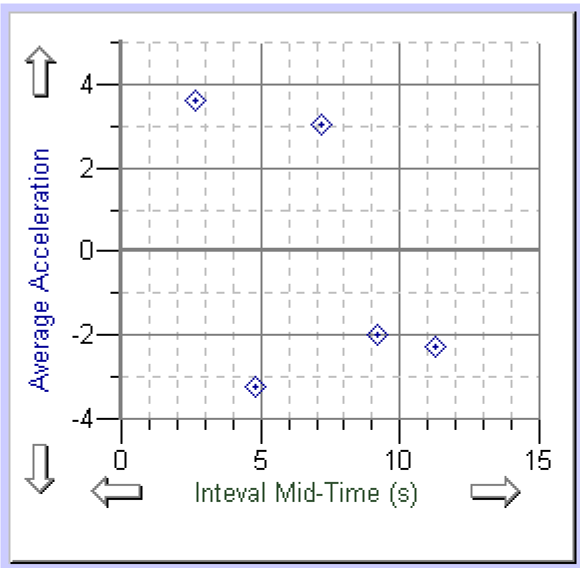
b) $a_{ave} = \text{slope} = (v_4 - v_1) / (t_4 - t_1) = (10.0 - 3.484848) / (8.25 - 1.65) =$ +0.987~~14~~ m/s²

c) $a_{ave} = \text{slope} = (v_6 - v_1) / (t_6 - t_1) = (1.20 - 3.484848) / (12.375 - 1.65) =$ –0.213~~039~~ m/s²

d) $a_{ave} = \text{slope} = (v_6 - v_5) / (t_6 - t_5) = (1.20 - 6.30769) / (12.375 - 10.125) =$ –2.27~~008~~ m/s²

Continuing this type of analysis with changing acceleration is considered an advanced topic. On most of the problems we look at in this course, the acceleration will be constant. Real life, however, is more like this example. Acceleration is only constant under certain conditions.

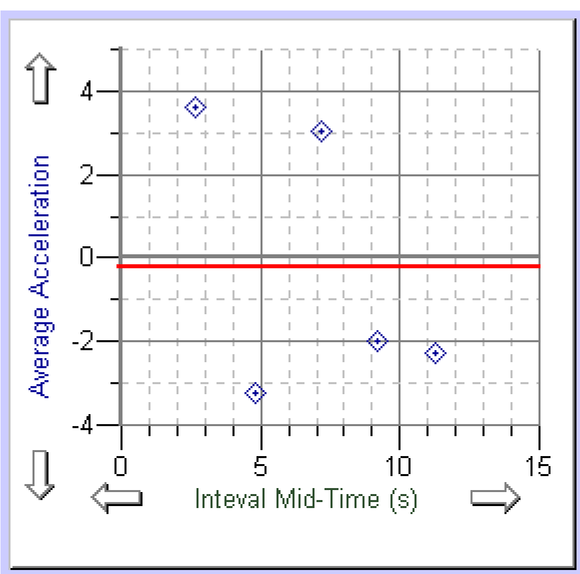
For this example, however, let's look a bit more carefully at the average acceleration between consecutive legs of the trip. You can work out the numbers for yourself. Just calculate the change in velocity and divide it by the time difference between the mid-point times for each leg. You will calculate five average accelerations working only between consecutive legs. Here is the graph of average acceleration between consecutive legs vs time.



This graph shows how the average acceleration jumps around from one segment of the trip to the next. One of the difficult questions we could ask ourselves is this: Is the data inconsistent with a constant acceleration or perhaps an acceleration of zero? Or, another way to look at that same question: Are the deviations from zero significant or are they the result of measurement errors?

One way to answer this question is to have a video recording of the motion, or a good memory of it. With a video recording we could perhaps re-measure the position vs time data with this specific question in mind. In the absence of a new look at the data, we will have to leave the question open. It does not look inconsistent with constant or zero acceleration, but there is no way to prove or disprove it without more information.

Unless we are sure the acceleration is constant, an open question in this case, we do not connect the points with lines. If the acceleration is constant, then we can draw a horizontal line. If we knew with some certainty that our data was taken on a system with a constant acceleration, then we would draw a horizontal line as shown in the final graph. The average acceleration for the whole trip is $-0.21\ m/s^2$. The horizontal red line represents this average acceleration for the whole trip.



A horizontal line on an acceleration vs time graph indicates constant acceleration. If the acceleration is constant and equal to zero, the horizontal line will coincide with the time axis on the graph. If the cart had decelerated at this rate for the entire time, it would have ended up reaching the same final velocity.