Dr. Mitch Hoselton 9/24/2003
Physics: An Incremental Development, John H. Saxon, Jr.

Lesson 09 – Torque – Introduction

The concept of torque arises from the need to simplify a certain class of force vs acceleration problems. The concept has merit for its simplifying prowess alone. However, I think most of us will agree there is something unsettling about spinning around in a circle that is different from the exhilaration that often comes from accelerating linearly. Spinning in a circle seems a bit too much like falling for some. It can be a bit scary and more than a bit confusing.

We should all remember that we are doing this for simplicity's sake and do our best to work through the complicated and scary bits to attain that goal.

One way to introduce torque is to imagine applying a uniform force to an object. When we've done this before, we got uniformly accelerated motion, i.e. constant acceleration in a straight line. If we apply that same force to an object that is nailed to the floor at one point, we get what seems to be an entirely different type of motion. First, call the nail a pivot and assume that no friction occurs at the pivot.

Under these circumstances the acceleration we expect is there, but it is acceleration around a circle. This type of acceleration has some unexpected quirks.

First, the values for instantaneous velocity and the acceleration you measure depend on how far you are from the pivot when you measure them. This is clearly going to be a messy system to understand.

Second, the values for the instantaneous velocity and the acceleration depend on both the mass and the shape of the object. This is really a mess. We can no longer rely on the force and mass to determine the acceleration. Somehow, we have to incorporate a shape factor into Newton's Second Law whenever the object is pivoted.

Third, to keep the object accelerating, we must constantly change the direction of the force. The optimum acceleration occurs when we keep the force perpendicular to a line connecting the pivot to the point where we are applying our force.

This is so complicated that we are not even going to try to work these problems using what we will now call the linear version of Newton's Second Law. We will, instead, jump right into the best possible means of working this type of problem; the rotational or angular version of Newton's Second Law. Like the linear version, it is simple to write down, but it takes some explanation and practice to learn how to use it properly. The angular version of Newton's 2nd Law is

$$\tau = I\,\alpha$$

Notice that each of the three variables is a counterpart to the three variables in the linear version of Newton's Second Law. Those similarities save the day for us.

Torque (τ) is the counterpart of force. Like force, it is a vector quantity. It points in a direction that is parallel to the axis about which the object is rotating. Since it must be parallel to the rotation axis, there are only two directions the torque vector can point. In each problem, we may arbitrarily assign one direction as positive and the other direction as negative. (See Lesson 35 for more on this view of torque.)

The moment of inertia (I) is the counterpart of mass. It is a scalar quantity. It includes the mass, but it also includes the distance to the pivot and a correction factor for the size and shape of the object. (See Lesson 58 for more on the moment of inertia.)

The angular acceleration (α) is the counterpart of acceleration. It measures the rotation in radians per second squared ($rad/s^2 = 1/s^2 = s^{-2}$) rather than in m/s^2. (See Lesson 25 for more on measuring angles in radians.)

Torque is not just Force

Torque is force applied at a distance from the pivot. The force must be applied at a distance otherwise it cannot rotate the object. If you think about it, even briefly, I think you will agree that you can push as hard as you like on the pivot and you will not be able to make the object rotate. Begin by asking yourself why door handles are placed far from the hinge. If the door handle was mounted on the hinge would that make it harder or easier to open a door?

As long as we agree that the force and the radial displacement, from the pivot to the point where the force is applied, are perpendicular to each other, the torque is simply the displacement times the force.

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

All three of the variables in this equation are vectors, so you will not be surprised to learn that a special type of multiplication is implied by the symbol, $\times$. We will learn more about the so-called cross-product or vector product in your next Physics course. For our purposes in this course, all you need to remember is that as long as we keep the **r** and **F** vectors perpendicular to each other, the vector-product gives exactly same result as normal multiplication of the magnitudes of the two vectors.

$$\tau = |\mathbf{r}| \cdot |\mathbf{F}_{\perp}| = r \cdot F_{\perp}$$

That **r** vector, frequently called the radius vector, does not provide information about the size of the object being rotated; only about the displacement of the force from the pivot point.

In this first introduction to torque, we are going to study the net torque under conditions where there is no net torque. When more than one force acts on an object, we may have more than one torque. The vector sum of the torques is the net torque.

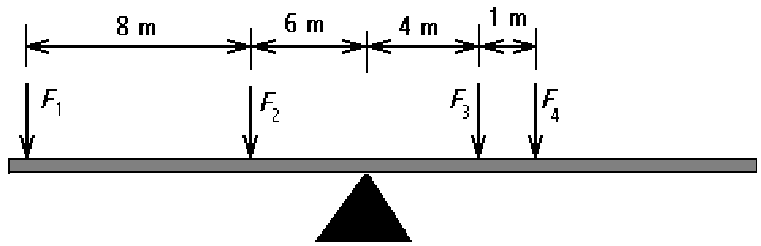
As with the linear form of Newton's Second Law, when more than one force acts on an object, we need to write our equation a bit differently; namely

$$\Sigma\tau = \tau_{Net} = \Sigma\,(r_i \cdot F_{\perp I})$$

Each force creates a torque, which must be given a sign; either positive or negative. We will arbitrarily assign positive signs to torques causing clockwise rotations and negative signs to torques causing counterclockwise rotations.

When the sum of the torques is zero there is no net torque and therefore no net angular acceleration. Likewise, when we view an object that is not exhibiting angular acceleration, we can be certain that it is not feeling a net torque. Here are some examples of massless rods subjected to two or more forces that produce no net torque.

Examples



I. *In this example, two of the forces will cause the massless beam to rotate in a clockwise direction. The other two forces will cause it to rotate in a clockwise direction. The beam is not rotating and three of the forces are known. Find the magnitude of the missing force.*

F_1, which is 14 m from the pivot point will cause a counterclockwise rotation. It produces a torque of $14 \cdot F_1$ N·m.

F_2, which is 6 m from the pivot point will cause a counterclockwise rotation. It produces a torque of $6 \cdot F_2$ N·m.

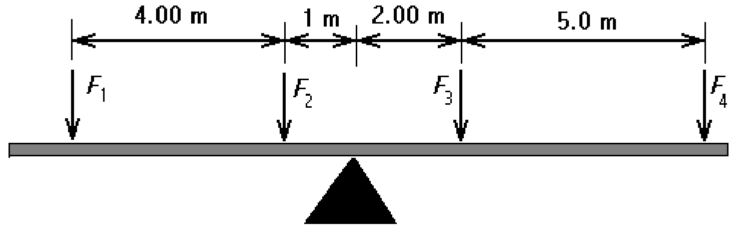
F_3, which is 4 m from the pivot point will cause a clockwise rotation. It produces a torque of $4 \cdot F_3$ N·m.

F_4, which is 5 m from the pivot point will cause a counterclockwise rotation. It produces a torque of $5 \cdot F_4$ N·m.

Therefore, $\Sigma\tau = \tau_{Net} = -\,14 \cdot F_1 - 6 \cdot F_2 + 4 \cdot F_3 + 5 \cdot F_4 = 0$

All these forces have positive magnitudes; only the torques they produce have positive and negative signs in this problem. The net torque is set to zero because the problem stipulates that the massless beam is not rotating. If it is not rotating, then it is certainly not undergoing a net angular acceleration. Therefore, the net torque must be zero. Given F_2, F_3, and F_4, you should be able to solve the equation for F_1.

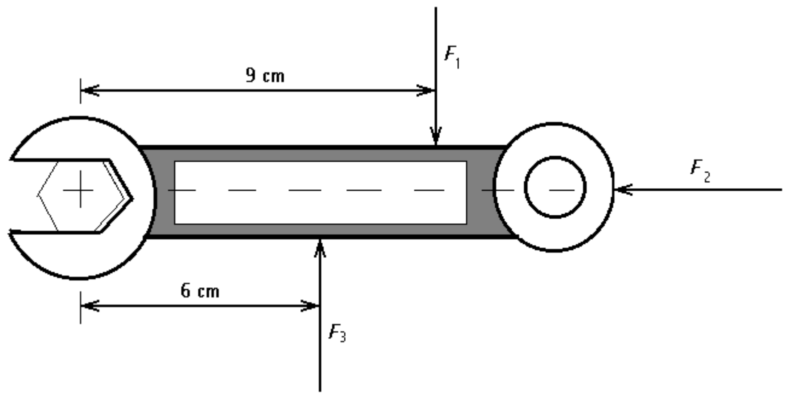
II. *This problem works just like the first one. Only the distances to the pivot have changed. Again, you are given three of the forces and the fact that the massless beam is not rotating. Find the magnitude of the missing force.*



This time the equation is $\Sigma\tau = \tau_{Net} = -5 \cdot F_1 - 1 \cdot F_2 + 2 \cdot F_3 + 7 \cdot F_4 = 0$

Given F_1, F_2, and F_4, you should be able to solve the equation for F_3.

III. *This problem only appears at first glance to be more complicated than the first two. In fact, we work it just like those other two problems. As long as it's not rotating, we will not need to worry about the moment of inertia and having an object with mass in the problem poses no new difficulty.*



The wrench is not rotating. Given F_1, find F_2 and F_3.

This problem does give us an opportunity to introduce a new term into your vocabulary. That is "moment arm." The moment arm is the shortest distance between the pivot point and the infinite, linear extension of the force vector's direction. Consider the force vector F_2. Its linear extension is an infinitely long, horizontal line that extends both left and right to infinity. That line, as it turns out in this case, passes through the pivot point. Therefore, the moment arm for F_2 is zero. The term "moment arm" is just another name for the radius vector perpendicular to the force vector. It is the **r** vector used to calculate the torque. The moment arms for the other two forces are given in the diagram.

The equation for the torque is $\Sigma\tau = \tau_{Net} = +9 \cdot F_1 + 0 \cdot F_2 - 6 \cdot F_3 = 0$

The value of F_2 cannot be determined. You could put any number into the equation. Since it will get multiplied by zero, its value disappears from the solution no matter what value to use. Given F_1, you should be able to solve the equation for F_3. Note that as long as all the distances are in the same units, we don't need to convert to meters to find F_3.