Lesson 23 - *Center of Mass*

It is the average position of mass in an object. On a predefined coordinate system, we find

In one dimension	$x_{CM} = \sum m_i x_i \ / \ Mass_{total}$

In two dimensions	$x_{CM} = \sum m_i x_i \ / \ Mass_{total}$
	$y_{CM} = \sum m_i y_i \ / \ Mass_{total}$

In three dimensions	$x_{CM} = \sum m_i x_i \ / \ Mass_{total}$
	$y_{CM} = \sum m_i y_i \ / \ Mass_{total}$
	$z_{CM} = \sum m_i z_i \ / \ Mass_{total}$

Properties:

1. If you apply a force only to the CM, it will cause no torque and the object will not rotate.

2. Applying the force of gravity to the CM is the same as applying it to the entire object;
   Therefore, falling objects experience no torque.

3. A force exerted on an object from inside the object does not change its motion.
   a) A cannon recoils when it fires a shell and the common center of mass does not move.
   b) The motion of the CM of a falling bomb does not change its motion after it explodes in mid-air.

4. If a force is exerted between two objects, their common CM does not change its motion.
   a) Planets and their moons always revolve around a common center of mass
   b) Binary stars revolve around a common center of mass.
   c) When objects collide, their common center of mass does not change its motion.

5. The center of mass is always <u>directly below</u> the pivot when an object is hung from a string.
   (This is called stable equilibrium; if you hang an object from several different points you can easily identify the one point that is directly below all of the hanging points. That point is the center of mass of the object.)

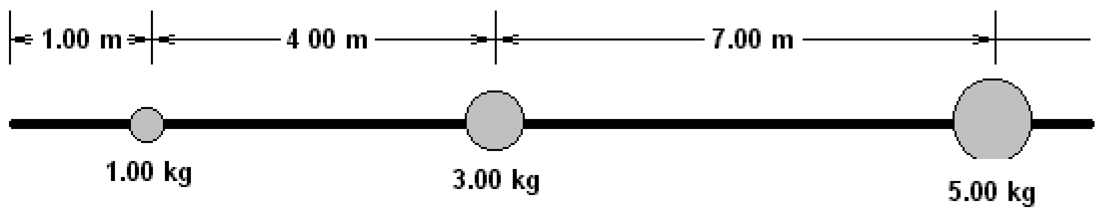
6. An object mounted with its CM above the pivot is unstable, because gravity will create a torque if the CM moves even a tiny distance away from the balance point. That torque will cause the CM to move even further from the balance point. The farther the CM is above a pivot point, the less stable the object is. (*Think about this when you think about high-speed turns and the stability of SUVs.*)

7. When you throw a tennis racket, or any irregular object, its CM traces a parabolic path. This is true no matter what wild rotations you impart to it as you launch it. The CM follows the same path a baseball would follow – a simple parabolic arc.

Examples

I. For "flat" one and two-dimensional objects, like meter sticks and plates, you can most easily find the center of mass by finding the balance point. Just balance it on the tip of your finger and the CM will be right above your fingertip.

II. For masses distributed along a massless rod, we use the one-dimensional equation. Let's say, in this example, that we want the distance from the left end of the rod to the CM. Label the left end the origin and use as our coordinates the distances of the masses from the left end.

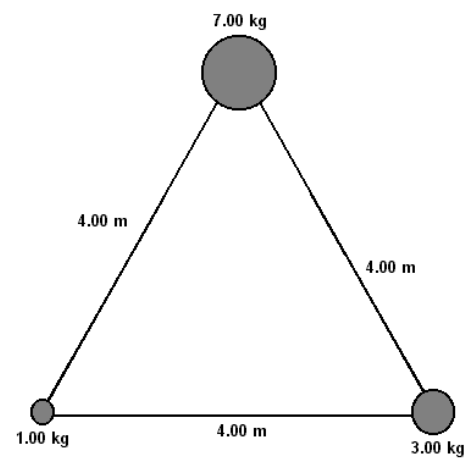


$x_{CM} = \sum m_i x_i$ / $Mass_{total}$ = [(1.00 x 1.00) + (3.00 x 5.00) + (5.00 x 12.00)] / (1.00 + 3.00 + 5.00)

= [1.00 + 15.00 + 60.00] / (9.00) = 76.00 / 9.00 = 8.44~~44~~ m (3 SF)

The CM is 8.44 m from the left end of the rod.

III. For masses distributed along a flat, massless wire frame, like these three masses at the corners of an equilateral triangular frame, we use the two-dimensional equations. In this case we will use the lower left-hand corner of the triangle as the origin.



$x_{CM} = \sum m_i x_i$ / $Mass_{total}$

= [(1.00x0)+(3.00x4.00)+(7.00x2.00)] / 11.00

= [0 + 12.00 + 14.00] / 11.00 = 2.3636

= 2.36~~36~~ m (3 SF)

$y_{CM} = \sum m_i y_i$ / $Mass_{total}$

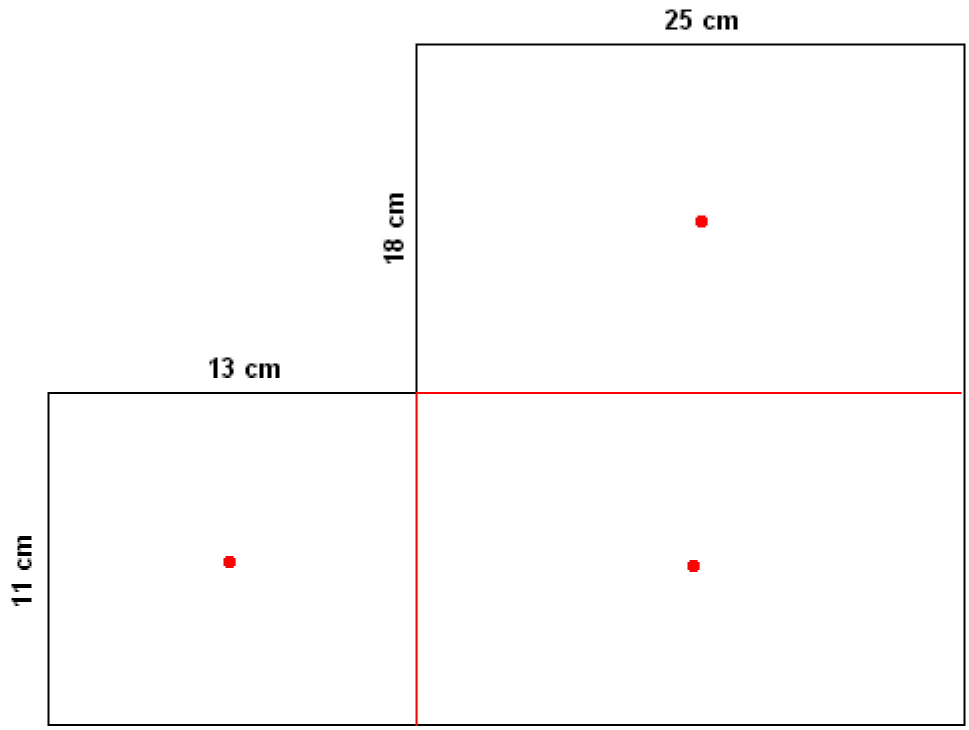
= [(1.00x0)+(3.00x0)+(7.00x$12^{1/2}$) / 11.00

= [0 + 0 + 24.248711] / 11 = 2.204428

= 2.20~~4428~~ m = 2.20 m (3 SF)

Can you identify this point on the diagram, at least, approximately? Recall that the origin is at the lower left hand corner of the triangle.

IV. For flat objects we use the two dimensional equations to find the coordinates of the CM. Use the lower left-hand corner of the object, or a point below and left of the object if it is highly irregular, as your origin. Identify the CM of each regularly shaped sub-part of the object (use squares and rectangles whenever possible, or triangles in a pinch). If the flat object has constant thickness and uniform density, we can substitute Area for Mass.



Using the lower left hand corner as the origin, the CMs of the three sub-parts I've marked out occur at

(6.5, 5.5), (25.5, 5.5), and (25.5, 20)

$x_{CM} = \sum m_i x_i \ / \ Mass_{total} = \sum A_i x_i \ / \ Area_{total}$ = [(143 x 6.5) + (275 x 25.5) + (450 x 25.5)] / 868

= [949 + 7,012.5 + 11,475] / 868 = 19,417.00 / 868

= 22.~~3698~~ cm = 22 cm (2 SF) from the left edge of the object

$y_{CM} = \sum m_i y_i \ / \ Mass_{total} = \sum A_i y_i \ / \ Area_{total}$ = [(143 x 5.5) + (275 x 5.5) + (450 x 20)] / 868

= [786.5 + 1,512.5 + 9,000] / 868 = 11,299.00 / 868

= 13.~~017281~~ cm = 13 cm(2 SF) from the bottom edge

The coordinates of the CM are (22 cm, 13 cm) relative to the lower left-hand corner. Can you find this point on the object – at least, approximately.