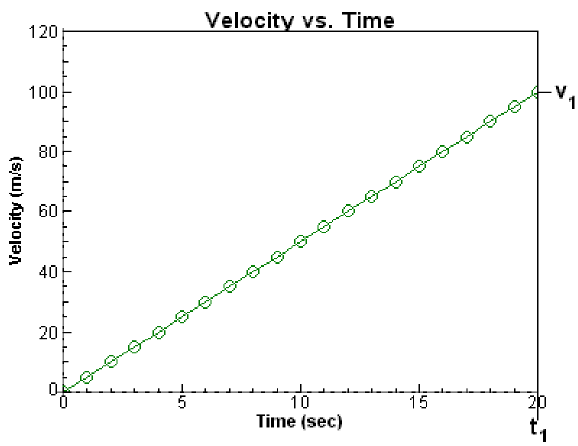
Lesson 24 – Galileo, Freely Falling Bodies & Uniformly Accelerated Motion

Galileo argued that a freely falling body is undergoing uniform acceleration. Its speed is increasing at a constant rate. The reason the speed increases at a constant rate is that a uniform force (*the "nearly" constant force of gravity*) is the only force acting on the body.

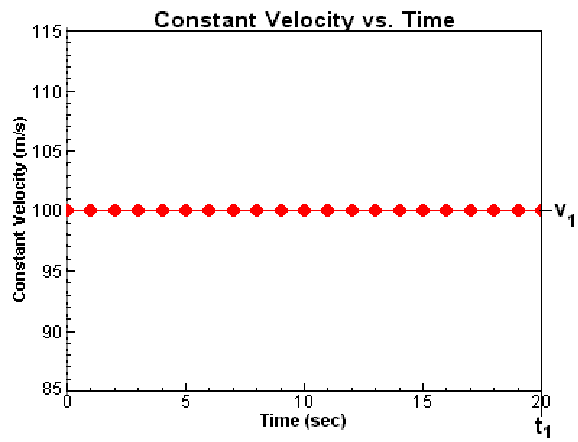
Compare these two graphs.

Accelerated motion



Slope of line = acceleration = a
Equation for the line is $v_t = a\ t$
v_o
Constant slope means constant a

Uniform Velocity



Slope of line = 0
Equation for the line is $v_t =$

Area under the curve represents the distance traveled ($x - x_o$).

$x - x_o = ½\ (base)(height)$
$(base)(height)$
$x - x_o = ½\ (t)(v_t)$
$x - x_o = ½\ (t)(a\ t)$
$x - x_o = ½\ a\ t^2$
(When v_o = 0 m/s and a = constant)

$x - x_o =$

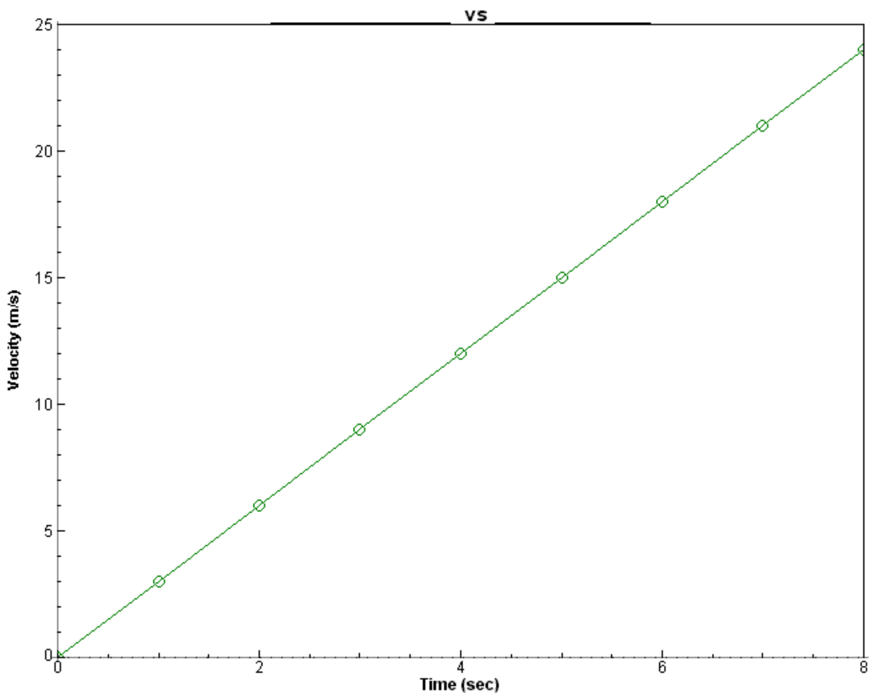
$x - x_o = (t)(v_o)$
$x - x_o = v_o t$
(When v_o = constant)

When the initial velocity is not conveniently zero, the equations of motion are all affected by this fact. What you get then is a combination of the two samples shown above. The distance equation works out to be the sum of two areas; like this.

$x - x_o = initial\ velocity\ x\ time + ½\ (base)(height)$
$x - x_o = v_o t + ½\ (t)(v_t - v_o)$
$x - x_o = v_o t + ½\ (t)(\Delta v_t)$
$x - x_o = v_o t + ½\ (t)(a\ t)$
$x - x_o = v_o t + ½\ a\ t^2$
(When $v_o \neq$ 0 m/s and a = constant)

We will work one of these below so you can see how to visualize this one on a graph.

Look at the example below. What does the graph compare? _______________ vs. __________



V vs t	
0 m/s	0 s
3	1
6	2
9	3
12	4
15	5
18	6
21	7
24	8

Is this accelerated motion? ________ How do you know?
____

Is the acceleration constant? _______ How do you know? ________________________

Find the slope of the line. ___________ What does the slope represent? ___________

Write a mathematical equation for the line above. ________________________
(*Use the slope-intercept form of a straight line equation*: y = b + mx)

What does the area under the line represent? _________________________

Distance moved during each second. (*Distance per second = m/s* ***or*** *average velocity*.)

_____	_____	_____	_____	_____	_____	_____	_____	D in m and v_{AVG} in m/s
1st	2nd	3rd	4th	5th	6th	7th	8th	

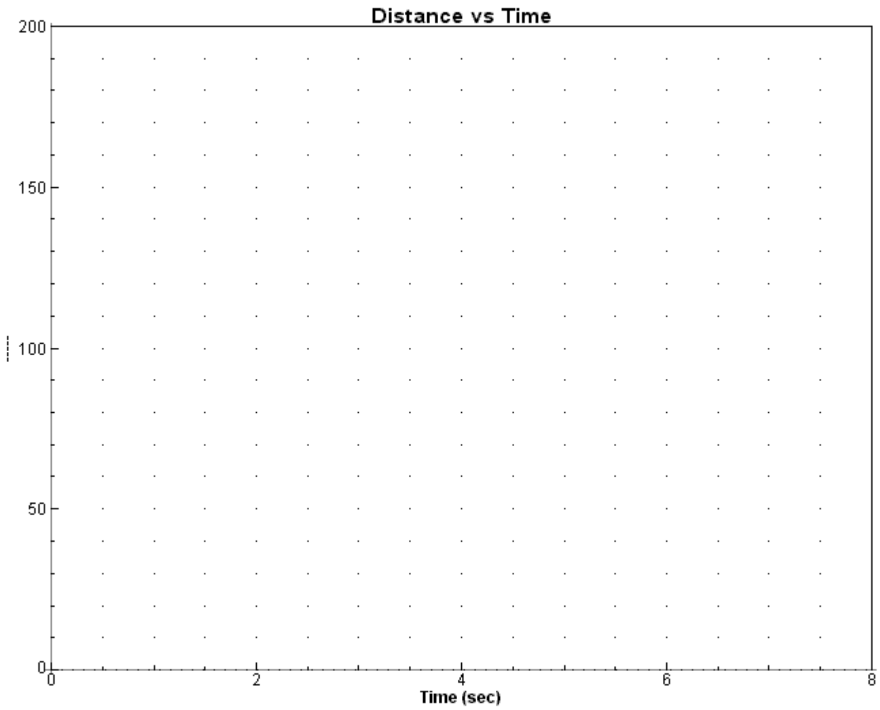
(*Plot these numbers in the middle of each second on the graph above.*)

Distance traveled from the origin by the end of each interval of the trip.

_____	_____	_____	_____	_____	_____	_____	_____
0 – 1 s	0 – 2 s	0 – 3 s	0 – 4 s	0 – 5 s	0 – 6 s	0 – 7 s	0 – 8 s

(*Notice that this distance is determined from the area of a triangle. The triangle has an area equal to one-half base times height* $\frac{1}{2}\ t\ (v) = \frac{1}{2}\ t\ (a\ t) = \frac{1}{2}\ a\ t^2$. *So the* *total distance traveled over any time interval can be calculated using:* $x - x_o = \frac{1}{2} a t^2$)

Now graph the distance traveled from the origin vs. time.



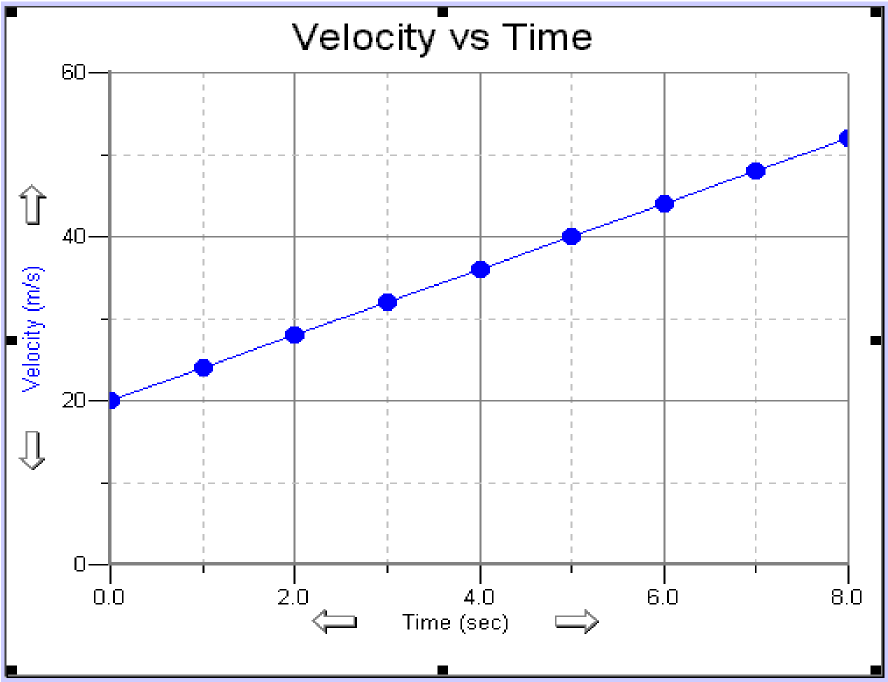
D	vs	t
_____		0 s
_____		1
_____		2
_____		3
_____		4
_____		5
_____		6
_____		7
_____		8

What is the name or shape of this curve? ____________________________________

What is the equation for this curve with symbols? ______________________________

Write the equation with numerical coefficients. _________________________________

Now suppose that the object starts with some speed ($v_o > 0$) and then accelerates.



V	vs	t
20 m/s		0 s
24		1
28		2
32		3
36		4
40		5
44		6
48		7
52		8

Is this accelerated motion? ________ How do you know?
____

Is the acceleration constant? _______ How do you know? ___________________________

Find the slope of the line. ____________ What does the slope represent? ____________

Write a mathematical equation for the line above. ___________________________
(*Use the slope-intercept form of a straight line equation*: y = b + mx)

What does the area under the line represent? ___________________________

Distance moved during each second. (*Distance per second = m/s* ***or*** *average velocity*.)

______	______	______	______	______	______	______	______	D in m and v_{AVG} in m/s
1st	2nd	3rd	4th	5th	6th	7th	8th	

(*Plot these numbers in the middle of each second on the graph above.*)

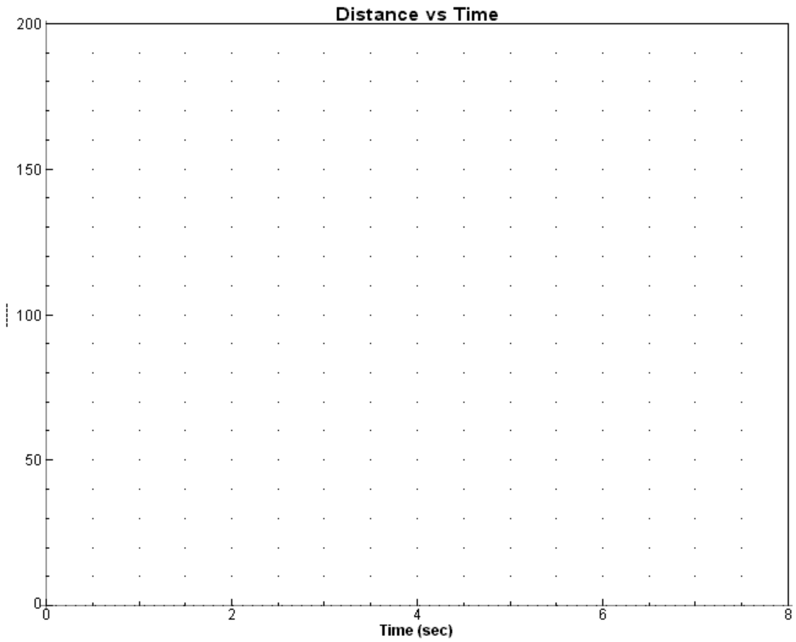
Distance traveled from the origin by the end of each second.

______	______	______	______	______	______	______	______
0 – 1 s	0 – 2 s	0 – 3 s	0 – 4 s	0 – 5 s	0 – 6 s	0 – 7 s	0 – 8 s

(*Notice that this distance can be broken into two areas: a rectangle with an area equal to* $v_o t$, *and a triangle at the top with an area equal to* $\frac{1}{2}\ t\ (\Delta v) = \frac{1}{2}\ t\ (a\ t) = \frac{1}{2}\ a$

t^2. T*he total distance traveled from the origin is calculated using:* $x - x_o = v_o t + \frac{1}{2} a t^2$)

Now graph the distance traveled from the origin vs. time.



D	vs	t
_____		0 s
_____		1
_____		2
_____		3
_____		4
_____		5
_____		6
_____		7
_____		8

What is the name or shape of this curve? ________________________________

What is the equation for this curve with symbols? ___________________________

Write the equation with numerical coefficients. ______________________________

Final Thought:

The concept that the area on a graph corresponds to a measurable quantity occurred first to Sir Isaac Newton in the course of developing the calculus. He began on page 1 of the first volume of his ***Principia*** expounding the means for determining the area between a curve and the bounds (axes) of the graph of that curve. He covered orbits, resistance, centripetal motion, pendulum motion and various types of forces. All this had been developed secretly because he considered it a type of "Cheat" to use these methods. You can see why. It is not enough to assert that the area is a measure of some quantity. You must prove it, if you want anyone to believe it. While Newton had no doubt, after all his private researches, that it was true, he nevertheless put off the task of proving it. It was not until he was threatened by the secretary of the Royal Society in London with the premature release of some of his results that he finally agreed to publish his methods and results. The ***Principia*** is the product of that publishing effort. It is not without considerable merit that the third volume of this work is entitled "The System of the World," by which Newton meant the "Universe." The ***Principia*** stands as an incomparable milestone in the development of scientific thought and method. It did indeed change the way we view the world. Even in this post-Einstein age, it still dominates the way most of us understand the universe on both large and small scales. Even Einstein had to acknowledge the fact that without a Newton, there could never have been an Einstein.