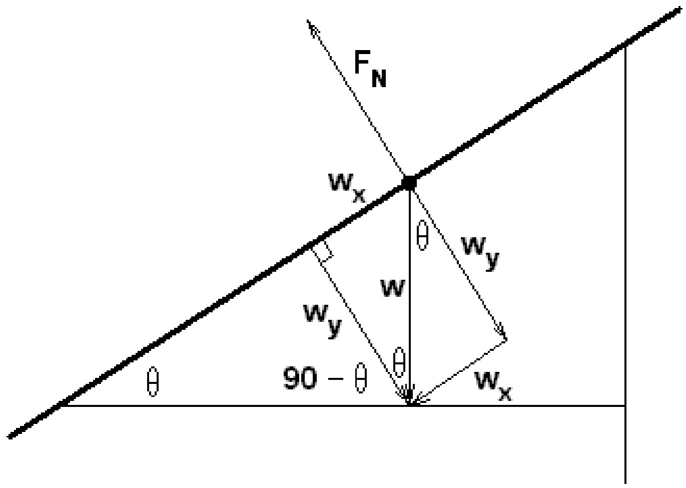
Chapter 27 – Inclined Plane without Friction

An object sitting on a tilted surface is subjected to at least two forces. One is gravity; it has weight. The other is the normal force exerted by the surface on the object. The weight, **w**, is always a vertical force. The normal force, $\mathbf{F_N}$, is always perpendicular to the surface. These two forces are shown in the following diagram.



If these are the only forces acting on the object represented by the spot, it will obviously slide spontaneously down the incline.

Let's take the two forces one at a time. $\mathbf{F_N}$ is perpendicular to the surface which means that it is also perpendicular to the direction the object will move as it slides down the slope. It is impossible for the normal force to have a component in the direction of the motion. Therefore, unless there is friction on the surface, the normal force does not contribute to the motion of the object.

The weight, **w**, on the other hand does have a component that points in the direction of the spontaneous motion of the object. The diagram shows the completed rectangle created by the two components of the weight. Specifically, it shows the two unique components that run parallel, $\mathbf{w_x}$, and perpendicular, $\mathbf{w_y}$, to the surface. The names $\mathbf{w_x}$ and $\mathbf{w_y}$ have been chosen to emphasize the fact that these two components are perpendicular to each other. Like the normal force, $\mathbf{w_y}$ is perpendicular to the spontaneous motion and cannot contribute to the motion of the object.

The only component, among the forces under discussion here, that can influence the motion is the one named $\mathbf{w_x}$. The key to working out the motion down the incline is finding the value of $\mathbf{w_x}$.

If you carefully consider the locations of the angle θ in the diagram, it should be clear that the magnitude of $\mathbf{w_x}$ equals **w** sin θ.

$$w_x = w \sin\theta$$

Newton's 2nd Law in the direction of the spontaneous motion then yields,

$$F = w \sin\theta = m\,a \quad \text{therefore} \quad [w \sin\theta] / m = a \quad \text{or} \quad a = [mg \sin\theta] / m = g \sin\theta$$

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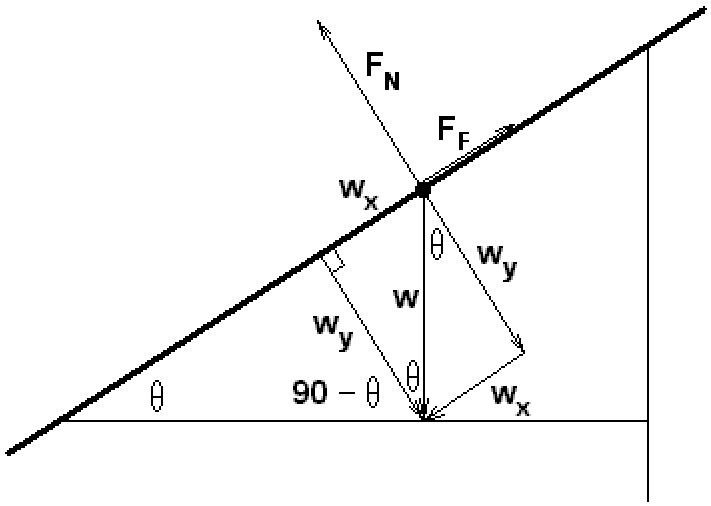
When there is friction in the problem, our force diagram for the inclined plane must be modified to include the additional force; as follows:



We can't be sure until we check by calculation whether the friction force is kinetic or static, so we will use the generic name $\mathbf{F_F}$ until we know for sure in each case. The friction force points opposite to the spontaneous direction of motion and will obviously have an influence on the motion. It might even prevent motion from occurring. We treat the direction of spontaneous motion as positive which means that the friction force will be treated as negative.

In order evaluate the friction force we will need to find the normal force. The only way to get at the normal force is to notice that the normal force must be equal to the component of the weight that is perpendicular to the inclined surface. In other words,

$$F_F = w_y = w \cos \theta$$

The general form of Newton's 2nd Law now looks like

$$\Sigma F_{EXT} = w \sin \theta - F_F = m\,a \quad \text{therefore} \quad [w \sin \theta - F_F] / m = a$$

There are only three possibilities:

I. The parallel component of the weight, $\mathbf{w_x}$, is greater than the friction force and the object accelerates down the incline. In this case, we are talking about kinetic friction, so

$$a = [w \sin \theta - F_K] / m = [w \sin \theta - \mu_K F_N] / m = [w \sin \theta - \mu_K\, w \cos \theta] / m$$

$$= [mg \sin \theta - \mu_K\, mg \cos \theta] / m = [g \sin \theta - \mu_K\, g \cos \theta]$$

$$= g\,[\sin \theta - \mu_K \cos \theta]$$

II. The maximum value of the static friction force is larger than the parallel component of the weight, $\mathbf{w_x}$. The static friction force is reactive and only gets large enough to exactly cancel the parallel component of the weight. It could get larger if we pushed harder or made the slope steeper. That means the object cannot slide even if we give it a small push.

There is no need to calculate the acceleration in this case. If the object is not moving it is also not accelerating, therefore,

$$a = 0$$

The only calculation we need to make is the one needed to determine which force is larger; $\mathbf{w_x}$ or $\mathbf{F_S}$. All we need to do is evaluate the expression in the square brackets, with μ_S this time. Thus,

If $[\sin\theta - \mu_S \cos\theta] < 0$ then $a = 0$ and the block is not moving.

If $[\sin\theta - \mu_S \cos\theta] > 0$ then $a > 0$ And we need to return to case **I** to find **a**.

III. For completeness sake, we also consider the rarest case, where the expression in square brackets equals zero. This means the object is poised on the verge of sliding down the incline. In fact, if we give it a small push, just to get it started, it will continue sliding down the incline at a constant speed without any further help from us. It will move without accelerating because the two forces acting on the object exactly cancel each other. The friction force acting on the object in this case is kinetic friction since the object is moving. Thus,

If $[\sin\theta - \mu_K \cos\theta] = 0$ then $a = 0$

Or, whenever $\mu_K = \tan\theta$ then $a = 0$

} And the object is moving with a small but constant velocity down the slope after being given a small push to get it started.

This is a familiar case. If the net force acting on an object is zero, then the object is at equilibrium. If the object is at equilibrium, then it is not accelerating. When an object is at equilibrium (not accelerating) then it may be moving at any constant velocity. Some of us forget that zero is not the only constant velocity. Any constant velocity is allowed for system at equilibrium.

Anytime we find an object sliding at constant speed, we can be sure the force(s) pointing against the direction of motion exactly cancel the force(s) pointing in the direction of motion.

$\mathbf{F_N}$ and $\mu_F \cos\theta$

What can we say about the normal force and the perpendicular component of the weight? It is most important to note that these two forces are equal and opposites of each other. How can we be sure? Because the object is not moving perpendicular to the surface. Our personal experience informs us that if we our little red wagon sits on the incline in the drive way, the little red wagon rolls along the incline to the end of the drive way. A little red wagon has never been seen to spontaneously float up off the driveway and accelerate into the air.

You can be sure that these two forces are equal and opposite in all cases. We use this fact frequently; every time we deal with inclined surfaces.