## Lesson 46 – Uniform Circular motion.

We use the term "uniform circular motion" to describe motion in a circle when the velocity vector tangent to the circumference of the circle has a constant magnitude. It does NOT have a constant direction, just a constant magnitude. If the object is a car, this means the speedometer will read a constant speed as the car goes around the circular track.

The direction of the velocity vector is constantly changing. Any change in the magnitude OR direction of a velocity vector is proof that a net force is being applied to the object and that the object is, therefore, accelerating. This is a little different from the type of acceleration that we usually envision, but it is acceleration nevertheless. When a vehicle moves at constant speed, but constantly changes its direction, it is accelerating just as surely as if it moved in a straight line but changed its speed.

**The type of acceleration that we can see when a dragster accelerates down a straight track and the type of acceleration we are discussing here seem very different to most of us. In spite of that seeming difference both "types" of acceleration are exactly the same and both obey Newton's Second Law of Motion.**

Before we get to the examples, let's look at the diagrams of these three factors: velocity, force and acceleration.

First, we'll look at each one separately. Then we will put them all together in one diagram so that you can see how they all fit together. One thing to remember is that there are three constants in this arrangement.

The magnitude of the velocity vector is constant.

The magnitude of the acceleration vector is constant.

The magnitude of the force vector is constant.

The direction of all three vectors is constantly changing, however. They change synchronously such that the object moves in a perfect circle with constant speed. All of this happens quite naturally and almost automatically because of the way the force acts on the object.

If is perhaps easiest to imagine, in an introduction to this type of accelerated motion, to imagine something very familiar. So, imagine that you have a mass on a string and that you are swinging it in a horizontal circle above your head. Tension in the string is the obvious force that keeps pulling the ball around in the circle. With a little more thought it should also be clear that the tension is pulling the ball toward the center of the circle. If fact, any constant force that pulls toward the center of a circle will work just as well.

Any force that pulls toward the center of a circle is called a centripetal force. That means "acting toward the center" or "center seeking." In the case of our mass on a string, the centripetal force is provided the tension in the string. In the case of a satellite orbiting the Earth, the centripetal force is provided by the gravitational force of the Earth pulling on the satellite, i.e. by the weight of the satellite. In the case of a car going around a corner, the centripetal force is provided by static friction between the tire and the road. The static friction prevents the tires from slipping sideways. It acts centripetally and pushes the car through an arc as it turns a corner or around an entire circle as it moves around a race track.

As you can see from these examples, centripetal force is not the name of a new force. It is an adjective used to describe a known force in this particular circumstance. One of the first skills you will need to master is an ability to find the force that is acting centripetally in each case.

You will see examples of same of the following forces acting as the centripetal force in specific cases in this course. A few homework problems covering specific examples are listed by chapter and problem number or Lab number for quick reference. This list is not complete. There are many more such problems in the textbook. Cases not covered in this course are described in parentheses.

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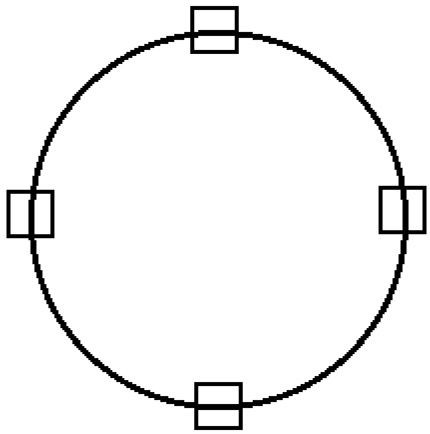
|---|---|
| **Weight**: 51-02 & orbits | **A component of the weight** (*Vertical circle, near but not at the top*) |
| **Normal force**: CB 55-05 | **A component of the Normal force**: 64-01 |
| **Friction**: 46-015, 5-01 & 02 | **A component of Friction** (*Fast on banked track*) |
| **Tension**: 49-05, & Lab 22 | **A component of the Tension**: 46-02, Lab 15 |
| **Magnetic force**: Lab 24 | **Electric force** (*electron orbiting a nucleus*) |
| **Buoyancy** (*circling in a maelstrom*) | **Spring force** (*Swing a mass on a spring*) |

That is a nearly exhaustive list. Expect to see most of these forces acting as the centripetal force at some point in the homework problems or labs. Be on the lookout for them in any problem where an object moves in a circle or through a circular arc at some point along its path. Whenever you see a circle or an arc, look for the force that is acting centripetally. If the question asks for the centripetal force, you will first need to identify the particular "type" of force, or forces, from among the forces in this list.

**The Diagrams**



Assume the object starts at the north position, distance $d_0$ = 0.00 m and time $t_0$ = 0.00 s.

We'll start with the basic diagram. This shows the position of the object at four different positions at four different times as it moves around the circle. The speed is constant. If T is the time it takes to make one full circle, then the snapshots take place at times $t_1 = T/4$; $t_2 = T/2$; $t_3 = 3T/4$ and $t_4 = T$ = the period.

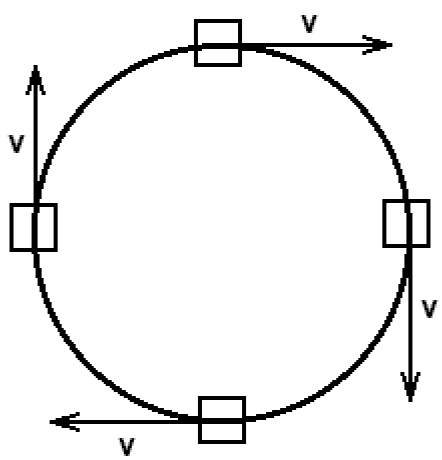
If C is the circumference of the circle, the object is shown at distances $d_1 = C/4$; $d_2 = C/2$; $d_3 = 3C/4$; and $d_4 = C$

The constant speed of the object can be calculated by dividing the distance traveled by the travel time. With a constant speed, it should not matter which location we use. Let's use them all.

$$\text{speed} = |\mathbf{v}| = (C/4)/(T/4) = (C/2)/(T/2) = (3C/4)/(3T/4) = (C)/(T) = C/T$$

We have already characterized these positions by the distance traveled and by the time taken to get there. We can also characterize each of these positions according to the instantaneous velocity vector, the instantaneous force vector and the instantaneous acceleration vector. So let's add these vectors singly to the basic diagram.

**The Velocity Diagram**

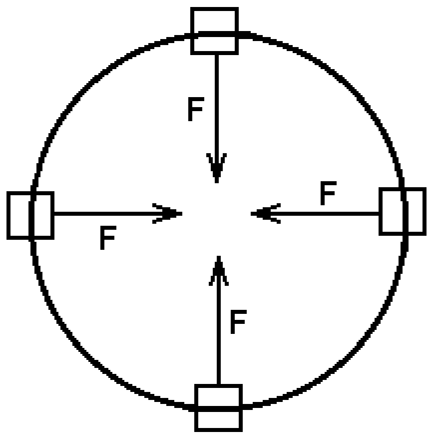



This is the diagram showing that the object moves at constant speed. Constant speed is exemplified by the fact that the instantaneous velocity vectors all have the same length (the same magnitude). New students, on first viewing such a diagram, may be tempted to think this problem is not related to Newton's Second Law. But there are two problems with that impression. Objects move in straight lines according to Newton's First Law unless a force acts on them. This object is clearly <u>not</u> moving in a straight line.

Second, if a force is acting on this object, it should be accelerating according the Second Law. So where is the acceleration. The acceleration is related to the change in direction; even though the speed is constant.

**The Force Diagram**

Without making too much of this little problem, it should be clear that we need a force pointing to the center of this circle in order to make the object move around this circle.




If you'll remember the example of a mass on a string, you will realize that the tension in that case is pulling the mass toward the center every instant while it rotates. If it stopped pulling to the center for even an instant, the object would fly off in a direction tangent to the circle.

The claim that the magnitude of the force, |**F**|, is constant is exemplified by the fact that the path of the object is a perfect circle. If the magnitude of the force varied even slightly, the circle would be distorted from its perfect shape.

As the object moves around the circle, the direction of the force must also keep changing in order to keep the object moving along the circle. If the force always pointed in the same direction the object would soon acquire a large velocity in the direction of the force and the circle would no longer be evident.

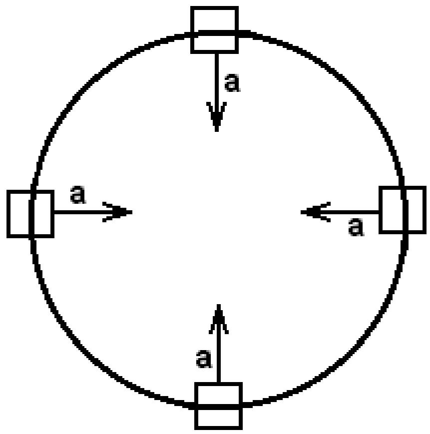
**The Acceleration Diagram**

There are three points to remember about the acceleration vector, **a**. First, acceleration is always in the direction of the force. As the force changes its direction, the acceleration must change its direction to match. The second is that the

$$\mathbf{a} = \mathbf{F}/m$$

For this reason we will draw the acceleration vector shorter than the force vector. This is merely a device to remind us this version of Newton's Second Law. Strictly speaking, the acceleration vector would be longer than the force vector any time the mass is less than 1.00 kg.

The third and very important point is that, at any given moment, the acceleration cannot be in the direction of the velocity vector. It cannot even have a component in the direction of the velocity vector. If the acceleration vector had a component in the direction of the velocity vector, that component would change the magnitude of the velocity vector. The velocity vector does not change its magnitude; it only changes direction. Clearly, the acceleration must point in a direction perpendicular to the velocity vector. Perpendicular and out of the circle will not work.



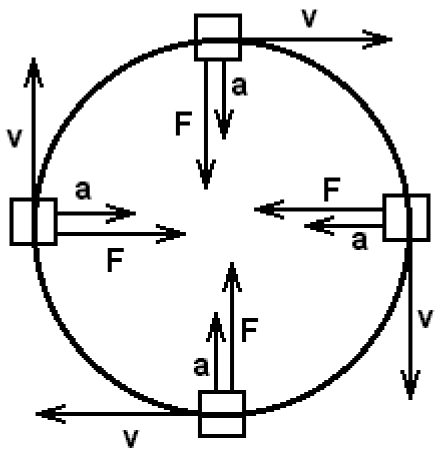

As we knew it would, the acceleration must point in the same direction as the force. Tht is perpendicular to the velocity and toward the center of the circle

The acceleration points in a direction that tracks the direction of the force.

If you think, again, in terms of the mass on a string, then we have to conclude that the force toward the center of a circle creates acceleration toward the center of the circle. That acceleration causes the object to move at constant speed in a perfect circle.

Acceleration toward the center of a circle or an arc always causes an object to move along that circle or arc.

**The Full Picture**




This diagram looks more complicated than we now know it to be. We've looked individually at all the parts of this diagram. Simply putting them all together doesn't change any of the meaning or understanding we've assembled. This diagram is simply a compact way to put it all together.

You can still break out the pieces in your imagination any time you need to think about just one aspect of this case. The most important diagram is, of course, the force diagram. Picture it without the velocity and acceleration parts of the diagram. That's the one you need to keep in mind when working a Second Law problem. The rest of this diagram should be helpful and now that you seen all the pieces, let's hope it will no longer seem so confusing.