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Physics: An Incremental Development, John H. Saxon, Jr.

Chapter 73 – Derivation of the Doppler Shift Equation

v_{SND} = constant relative to the ground <u>or</u> relative to a person or detector not moving with respect to the ground. Calculating the speed of sound is covered in a separate handout.

v_O and v_S, speeds of the observer and source, respectively. Consider yourself the observer and the source is the source of sound; e.g. car horn, ambulance siren, train whistle, etc. We assume that both the SRC and OBS move at constant velocities, at least for the duration of our calculations.

f_0 and f' are the base frequency and the Doppler shifted frequency detected by the observer.

For all cases we assume straight-line motion along a common axis (i.e., this is a one-dimensional problem; both SRC and OBS are sitting on or moving along the same straight road, straight railroad track, etc.).

For this derivation we first consider a stationary observer with the source moving <u>toward</u> the observer. Here is a simplified diagram showing the crest of the sound wave emitted from the moving source, which is about to emit the next crest.

We begin with the basic wave equation relating frequency, wavelength and velocity. Then we proceed with simple algebra, as follows

$$f' = \frac{v_{SND}}{\lambda_0 - \Delta\lambda} = \frac{v_{SND}}{\lambda_0 - v_S T} = \frac{v_{SND}}{\lambda_0 - v_S / f_0} = \frac{v_{SND}}{\frac{\lambda_0 f_0}{f_0} - \frac{v_S}{f_0}} = \frac{v_{SND}}{\frac{v_{SND}}{f_0} - \frac{v_S}{f_0}} = \frac{v_{SND}}{\frac{v_{SND} - v_S}{f_0}}$$

$$= f_0 \frac{v_{SND}}{v_{SND} - v_S}$$ **negative (-) when source moves TOWARD observer.**

Notice that the source velocity is in the denominator and that it is subtracted when the source moves toward the observer. These are key features of our equation in the Formula Sheet. This makes sense. Subtraction makes the denominator smaller which makes the ratio larger which indicates an increase in frequency. That is exactly what the diagram shows. The wavelength is getting shorter and the frequency is increasing.

Next we consider a stationary observer with the source moving away from the observer. Here is a simplified diagram showing the crest of last sound wave emitted from the moving source, which is about to emit the next crest.

As before, we begin with the basic wave equation relating frequency, wavelength and velocity. Then we proceed with simple algebra, as follows

$$f' = \frac{v_{SND}}{\lambda_0 + \Delta\lambda} = \frac{v_{SND}}{\lambda_0 + v_S T} = \frac{v_{SND}}{\lambda_0 + v_S / f_O} = \frac{v_{SND}}{\frac{\lambda_0 f_O}{f_O} + \frac{v_S}{f_O}} = \frac{v_{SND}}{\frac{v_{SND}}{f_O} + \frac{v_S}{f_O}} = \frac{v_{SND}}{\frac{v_{SND} + v_S}{f_O}}$$

$$= f_O \frac{v_{SND}}{v_{SND} + v_S}$$ **positive (+) when source moves AWAY from observer**

Notice that the source velocity is still in the denominator and that it is added when the source moves away from the observer. These are key features of our equation in the Formula Sheet. This makes sense. Addition makes the denominator larger which makes the ratio smaller which indicates a decrease in frequency. That is exactly what the diagram shows. The wavelength is getting longer and the frequency is decreasing.

Finally, we consider, what effect motion by the observer has on the apparent frequency of the observed sound. If the observer moves toward the source then it will encounter the crests of the wave more frequently, at higher frequency, while if it moves away from the source it will encounter the crests less frequently, at lower frequency. In other words, the effect of the observer's motion is to change the apparent speed of sound.

We can incorporate the effect of a moving observer by replacing the speed of sound in the numerator with the apparent speed of sound. The apparent speed of sound is simply the relative speed of the observer and the sound wave.

The relative speed is $v_{SND} + v_s$ when the observer is moving toward the source of the sound. The relative speed is $v_{SND} - v_s$ when the observer is moving away from the source of the sound. Therefore; if $v_{SND} = 343$ m/s

$$f' = f\,\frac{343 \pm_{Away}^{Toward} v_o}{343 \mp_{Away}^{Toward} v_s}$$