Chapter 73 - Doppler Shift – Changes in Sound Frequency

S_{SND} = constant relative to the ground <u>or</u> relative to a person or detector not moving with respect to the ground. Calculating the speed of sound is covered in a separate handout.

v_O and v_S, speeds of the observer and source, respectively. Consider yourself the observer and the source is the source of sound; e.g. car horn, ambulance siren, train whistle, etc. We assume that both the SRC and OBS move at constant velocities, at least for the duration of our calculations.

f_0 and f' are the base frequency and the Doppler shifted frequency heard by the observer.

Not counting the trivial case, where neither is moving, ten (10) cases are considered. These ten cases constitute a complete analysis of this type of problem.

<u>Only one is moving</u>

1. SRC moving toward OBS & OBS not moving
2. SRC moving away from OBS & OBS not moving
3. OBS moving toward SRC & SRC not moving
4. OBS moving away from SRC & SRC not moving

<u>Both are moving, but in opposite directions</u>

5. SRC and OBS moving toward each other
6. SRC and OBS moving away from each other

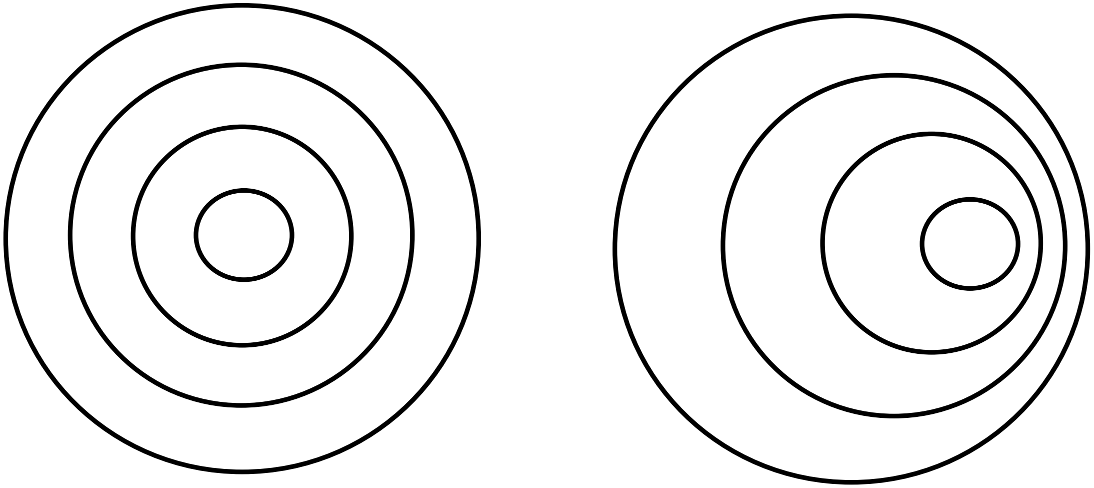
<u>Both are moving in the same direction</u>

7. SRC behind OBS, SRC moving faster and catching up
8. OBS behind SRC, SRC moving faster and pulling away
9. OBS behind SRC, OBS moving faster and catching up
10 SRC behind OBS, OBS moving faster and pulling away

For all cases we assume straight-line motion along a common axis (i.e., this is a one-dimensional problem; both SRC and OBS are sitting on or moving along the same straight road, straight railroad track, etc.).

1st key step in the solution: *i*) is the SRC moving toward (–) or away (+) from the OBS?
***ii*) is the OBS moving toward (+) or away (–) from the SRC?**

For a stationary source, the sound waves expand uniformly in all directions from the source. This produces a circular profile of wave crests as shown in the left half of the following diagram. We are looking only at the crests of the waves, but we could pick any convenient part of the wave and we would get the same diagram. The crest that was emitted earliest is farthest from the center. The most recently emitted crest is nearest the center. All the crests expand outward at the same constant speed of sound. The radial distance between successive crests is the wavelength of the sound waves, λ.



On the other hand, when the source is moving, successive crests are emitted from different centers. On the right side of the diagram, we see an exaggerated example of wave crests emitted by a source that is moving almost as fast as the speed of sound. The source is moving fast enough that the next emitted crest is close behind the last emitted crest in the direction of motion. Of course, in the opposite direction the crests are farther apart than they were in the case of the stationary source.

This bit of understanding is key to the analyses that follow. Remember that a wave emitted by a moving source appears to be either wavelength compressed or wavelength stretched depending on where you stand with respect to the source.

The other bit of understanding you need to remember is the wave equation for the speed of sound. Speed equals frequency times wavelength, and the speed of sound is a constant.

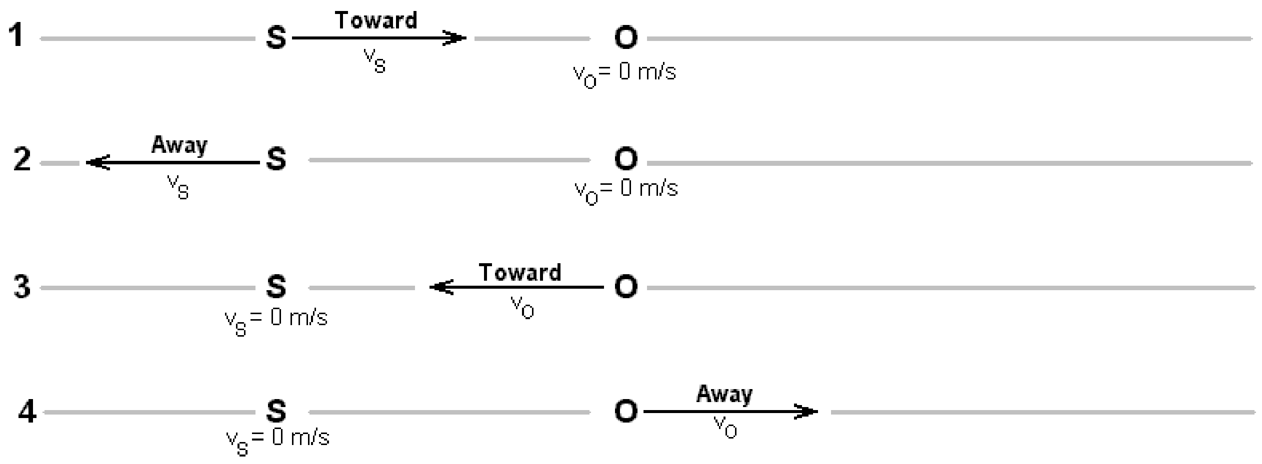
$$S_{SND} = f\lambda = \text{constant}$$

Because the speed is constant, anything that changes the wavelength also changes the frequency in the opposite sense. For example, if it increases the wavelength, then it must decrease the frequency in order for the speed to remain constant.

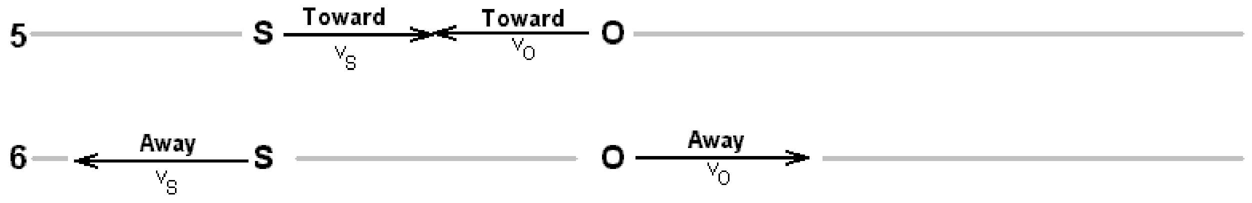
We are only discussing the frequency and wavelength directly ahead or directly behind the moving source. We are not going to discuss the continuous range of frequencies that we would detect if we were standing at locations off axis to the motion. Even with this simplification there are still ten cases we need to discuss.

Here is a diagrammatic presentation of the ten cases listed on the first page.

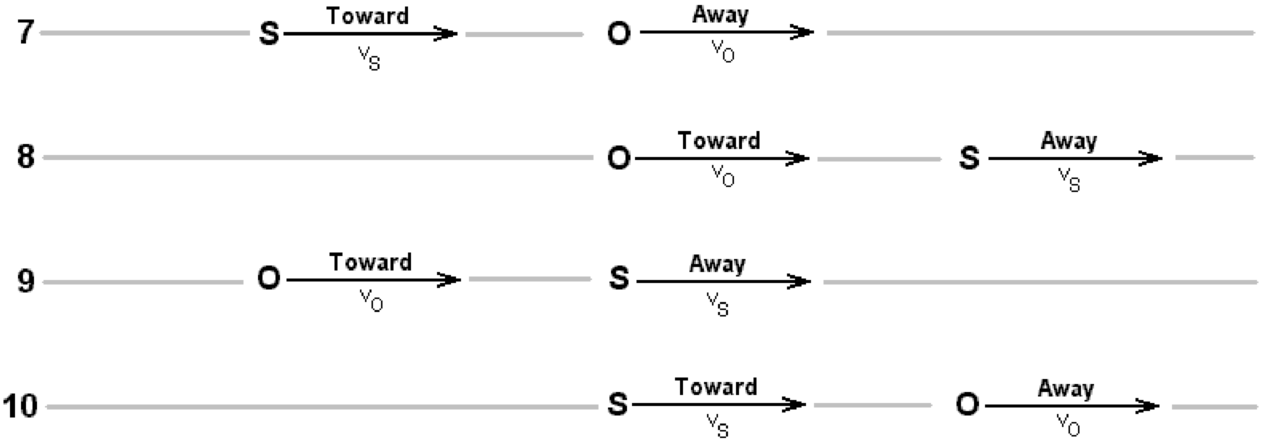
Only one is moving



Both are moving, but in opposite directions



Both are moving in the same direction



As we work through these cases assume the faster one moves at v = 45 m/s and the slower one moves at v =15 m/s. We will use s_{SND} = 343 m/s for the speed of sound in these examples. Our reference frequency will be 500 Hz, which is the frequency that a stationary observer hears when observing a stationary source.

Your formula sheet gives the following formula to be used for calculating the Doppler shifted frequency in the 10 cases.

Doppler Effect

$$f' = f\,\frac{343 \pm^{Toward}_{Away} v_o}{343 \mp^{Toward}_{Away} v_s}$$

v_o = velocity of observer: v_s = velocity of source

The signs in front of the source and observer velocities are determined according whether movement of one is toward or away from the other. For example, the observer velocity is added if it is moving toward the source, while the source velocity is subtracted if it is moving toward the observer. For motion away the signs switch.

Here are the correct answers for the 10 trial scenarios. Verify that you can get the same answers if you use the given velocities, directions, and base frequency. The practice will be both useful and instructive.

1. **$v_O = 0.0$ m/s** **$v_S = 45.0$ m/s** toward(–)

f = 500 Hz; *f’* = 576 Hz

Δ*f* = +76 Hz

2. **$v_O = 0.0$ m/s** **$v_S = 45.0$ m/s** away(+)

f = 500 Hz; *f’* = 442 Hz

Δ*f* = –58 Hz

3. **$v_O = 15.0$ m/s** toward(+) **$v_S = 0.0$ m/s**

f = 500 Hz; *f’* = 522 Hz

Δ*f* = +22 Hz

4. **$v_O = 15.0$ m/s** away(–) **$v_S = 0.0$ m/s**

f = 500 Hz; *f’* = 478 Hz

Δ*f* = –22 Hz

5. **$v_O = 15.0$ m/s** toward(+) **$v_S = 45.0$ m/s** toward(–)

f = 500 Hz; *f’* = 601 Hz

Δ*f* = +101 Hz

6. **$v_O = 15.0$ m/s** away(–) **$v_S = 45.0$ m/s** away(+)

f = 500 Hz; *f’* = 423 Hz

Δ*f* = –77 Hz

7. **$v_O = 15.0$ m/s** away(–) **$v_S = 45.0$ m/s** toward(–)

f = 500 Hz; *f’* = 550 Hz

Δ*f* = +50 Hz

8. **$v_O = 15.0$ m/s** toward(+) **$v_S = 45.0$ m/s** away(+)

f = 500 Hz; *f’* = 461 Hz

Δ*f* = –39 Hz

9. **$v_O = 45.0$ m/s** toward(+) **$v_S = 15.0$ m/s** away(+)

f = 500 Hz; *f’* = 542 Hz

Δ*f* = +42 Hz

10. **$v_O = 45.0$ m/s** away(–) **$v_S = 15.0$ m/s** toward(–)

f = 500 Hz; *f’* = 454 Hz

Δ*f* = –46 Hz